TE MANU KA RERE

NCEA Targeted Support for Secondary Schools

Targeted Support for Numeracy Intervention Planning

Overview

What: An approach to targeted support for Numeracy learning for small groups, to support accelerated learning progress, and achievement in the NCEA Numeracy common assessment activity (CAA) co-requisite assessments.

How: A 10-lesson programme, 1-hour per lesson for small groups (approx. 5 students per group) supporting accelerated learning of key numeracy skills.

Outcomes:

1. Targeted students are identified using readiness indicators, including e-asTTle and teacher observations, and targeted support is implemented to accelerate learning progress for these students in small groups;
2. Teachers are provided with appropriate professional learning, detailed guidance and develop capacity to provide targeted support and accelerated learning opportunities; and
3. Targeted students demonstrate accelerated learning progress and achievement in the NCEA Numeracy common assessment activity co-requisite assessments.

Note: These lessons and activities are intended as a guide only and this planning should be adapted to context and learners, using contextually relevant topics/ themes and examples – to engage and meet learners' needs.

Additional practice for each concept would also be beneficial, using relevant questions from the past CAA papers. This would reinforce the learning and provide practice in the context of the CAA.

This *Targeted Support for Numeracy Intervention Planning* document includes the following:

1. *Targeted Support for Numeracy Intervention Planning* overview
2. Key terminology
3. NCEA numeracy information
4. NCEA numeracy learning matrix
5. Targeted Support for Numeracy Intervention Plan, including a teaching and learning plan overview and detailed lesson plans (10)
6. Additional support resources

Key terminology

The following key terminology supports the understanding of the focus of this intervention.

Acceleration: In the current New Zealand Curriculum[i], students are generally expected to progress through one curriculum level every two years.[ii] This means students would typically be expected to have mastery of Level 5 (or Phase 4 of Te Mātaiaho: The Refreshed New Zealand Curriculum) and be ready to operate at Level 6 (or Phase 5 of Te Maātaiaho: The Refreshed New Zealand Curriculum) by the end of Year 10 (in preparation for NCEA Level 1 in Year 11). However, individual student progress may vary, and some students may progress faster or slower than this general guideline

Accelerated learning progress: Accelerated learner progress refers to: "... advancing the learning of children related to content at their current year level. Accelerated learning is achieved through specific teaching strategies, learning conditions, or scaffolded supports that enable learners to acquire skills more rapidly than they would under 'usual' teaching conditions. It relies on continuous monitoring of student progress against benchmarks and progress indicators, ensuring that the learning process remains aligned with year-level expectations." (Gillon et al, 2024.)[iii]
Accelerated learning progress means students make more than one year's worth of progress in a year. This also includes progress that is noticeably faster than expected based on their previous learning, as assessed by tools that measure reading, writing, and math skills.[iv]

Numeracy: Numeracy is a foundational skill that enables access to further learning, develops important life skills, and allows people to fully engage in work and in their communities.[v]

Targeted support: Targeted support builds on 'Universal' (tier 1) classroom teaching, accelerating the progress of students needing extra help so they can fully engage with their year level curriculum. Effective universal classroom teaching that includes small group work targeted to specific needs is part of the same continuum as targeted teaching.
Targeted support including small group work can be integrated into the regular classroom programme and does not require students to be removed from their regular classroom.
Targeted teaching accelerates progress through focused small-group instruction. It is additional, explicit instruction that models skills, addresses specific needs, and helps students apply learning in new contexts – it's not just more lessons. Effective targeted teaching reflects students' cultural identities, languages, knowledge, beliefs, and experiences.[vi]

NCEA Numeracy

Whiria te kaha tūātinitini, whiria te kaha tūāmanomano

This whakatauākī speaks to the many threads – academic, social, emotional, and cultural – that learners experience when they make mathematical and statistical decisions about situations in their daily lives. These threads include the satisfaction learners experience when they use mathematics and statistics to understand situations; the joy in using mathematical and statistical ideas to improve the lives of others; and the experience of mathematics and statistics as creative and empowering human endeavours.[vii]

NCEA Numeracy co-requisite common assessment activity (CAA) assessments[viii]

- All learners need to achieve a 20-credit co-requisite specific to te reo matatini or literacy and pāngarau or numeracy skills to be awarded any level of NCEA. The co-requisite is a one-off requirement.
- The requirements are formally separated out from the certificate at Levels 1-3 and the 20-credit co-requisite becomes mandatory from 2024.
- The transition period for the implementation of the NCEA Co-requisite, originally scheduled to end in 2025, has been extended by another two years up to the end of 2027. This will give schools, kura, and all NCEA providers extra time to adjust to the new requirements and strengthen the teaching and learning of te reo matatini, pāngarau, literacy and numeracy.
- From 2028, dedicated standards, or common assessment activities (CAAs) for te reo matatini, pāngarau, literacy, and numeracy will be the only method to achieve the NCEA Co-requisite.

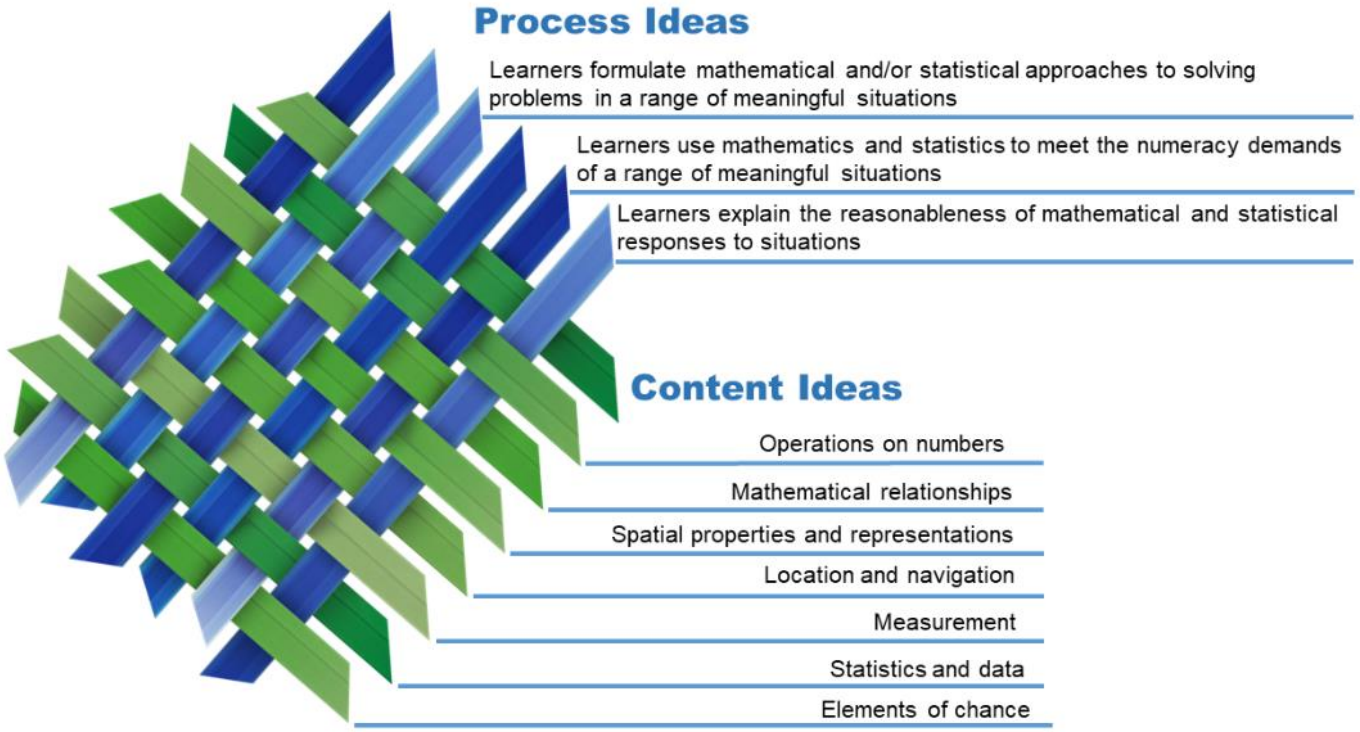
The NCEA Numeracy Pāngarau common assessment activities (CAAs) for assessment of the co-requisite requirements are:

- **Numeracy** US32406 (v3): *Apply mathematics and statistics in a range of everyday situations;*
  Purpose: Learners credited with this unit standard are able to formulate mathematical and statistical approaches to solving problems in a range of everyday situations, use mathematics and statistics to meet the numeracy demands of a range of everyday situations, and explain mathematical and statistical responses to situations.
  Level: 1 **Credits**: 10

- **Pāngarau** US32412: *Te whakamahi pāngarau hei whakaoti rapanga o te ao o te ākonga*.
  Whainga: Ko ngā ākonga kua whakawhiwhia ki te paerewa paetae nei, e āhei ana ki te whakamahi pāngarau hei whakaoti rapanga o te ao o te ākonga.
  Kaupae: 1 **Whiwhinga**: 10

The Numeracy standard requires ākonga to master the mathematics and statistics content ideas at Level 4 of the New Zealand Curriculum AND interweave these content ideas with mathematical and statistical process ideas.

Weave diagram for numeracy illustrating process and content ideas[ix]



NCEA Numeracy Learning Matrix

NUMERACY (US32406) *Apply mathematics and statistics in a range of everyday situations*		
Process Ideas	**Significant Learning** *Learners…*	**Outcomes and performance criteria**
Learners formulate mathematical and/or statistical approaches to solving problems in a range of meaningful situations.	• determine the mathematics and/or statistics needed in a range of situations. • formulate plans to use mathematics/statistics • select appropriate representations of the mathematics or statistics – e.g. graphs, tables, diagrams, equations, expressions.	**Outcome 1**: Formulate mathematical and statistical approaches to solving problems in a range of everyday situations. **Performance criteria** 1.1 Select an appropriate operation, representation, variable, and/ or method to solve the problem(s).
Learners use mathematics and statistics to meet the numeracy demands of a range of meaningful situations.	• apply mathematical and/or statistical concepts. • use appropriate mathematical/statistical approaches, which may include digital calculations. • • use a degree of precision appropriate to the situation (including estimation).	**Outcome 2**: Use mathematics and statistics to meet the numeracy demands of a range of everyday situations. **Performance criteria** 2.1 Apply mathematical and statistical procedures correctly in the situations.

Learners explain the reasonableness of mathematical and statistical responses to situations.	• consider and explain the reasonableness of solutions, outcomes, and approaches while reflecting on how these were chosen.
	• engage in sense-making to interpret solutions in relation to the situation given, including in different cultural contexts (see "Unpacking Numeracy").
	• provide evidence-based conclusions.
	• use critical judgements in relation to statements based on mathematical and statistical ideas.
	  ○ critique these statements
	  ○ explore different approaches to them
	  ○ respect the thinking of others
	  ○ respond to the ideas of others
	• share mathematical and statistical ideas.
	  ○ use mathematical and statistical language, symbols, and representations
	  ○ share methods and results
	  ○ use their own words to explain ideas
	  ○ present ideas concisely and coherently
		Outcome 3: Explain mathematical and statistical responses to situations
		
		Performance criteria
		3.1 Use evidence to explain the selection of the method and/or calculation.

Content ideas

1. Fluently and flexibly solve problems that require operations on numbers, understanding the relative size of those numbers, and making sense of the answer in context.
2. Recognise and work with mathematical relationships.
3. Understand and use the spatial properties and representations of objects.
4. Understand and use systems for location and navigation.
5. Use numbers and units to measure and express attributes of objects and events as quantities, with a degree of precision appropriate to the context.
6. Understand and reason with statistics and data.
7. Use probability to interpret situations that involve elements of chance.

Targeted support for accelerating numeracy intervention plan

The following **10-lesson teaching and learning plan** is designed to support teaching and learning for the **NCEA Numeracy Co-requisite** assessments focusing on key numeracy skills for **US32406: Apply mathematics and statistics in a range of everyday situations**. The plan integrates key **acceleration strategies** to support learners who need scaffolding and targeted support to meet the numeracy standard.

Key acceleration strategies for NUMERACY learning

- **ASSESSMENT FOR LEARNING: Formative assessment**
  Clear learning intentions and success criteria with frequent check-ins and self-assessment. Regular formative assessments and progress checks help you understand where students are at and what they need next, and to uncover misconceptions. Use frequent and timely feedback and feedforward.
- **COLLABORATIVE LEARNING: Co-operative learning**
  Create opportunities for small group work and peer-assisted learning, where students can talk through their thinking and learn from each other.
- **CULTURAL RESPONSIVENESS: Culturally responsive pedagogy**
  Create environments that respect and affirm students' cultural identity. Use authentic, relevant contexts for learning.
- **DEVELOPING MATHEMATICAL VOCABULARY: Mathematical language routines (MLR)**
  Teach precise mathematical vocabulary with clear definitions and encourage its use in discussions to help students communicate mathematical ideas effectively. Use mathematical language routines to reinforce mathematical language learning.
- **EXPLICIT TEACHING**
  Explicit teaching in relevant and engaging situations that encourage a positive relationship with mathematics and statistics. Focus on essential learning by teaching critical skills and concepts rather than re-teaching everything.
- **FOSTERING ENGAGEMENT**
  Build confidence through positive learning and experiencing success. Generate thinking, purpose, relevance, and curiosity. Foster positive mathematical identities. Engaging, hands-on learning to help students retain and apply concepts and procedures.
- **SCAFFOLDING: Chunking and modelling**
  Breaking down complex tasks and showing "how-to" steps.
- **SCAFFOLDING: Scaffolded learning**
  Provides and gradually removes supports as learners gain independence. Encourage students to use problem-solving strategies so they feel confident tackling challenges.

The Targeted Support for Numeracy Intervention Planning Supplementary PowerPoint is used throughout this plan and includes examples of previous assessment questions.

Teaching and Learning Plan Overview

Lesson	Focus	Overview of activities	Acceleration Strategies
1	**Understanding the numeracy CAA**	1. Vocabulary Builder 2. Exploring the 'Process Outcomes' 3. Exploring the 'Content Areas'	• Developing mathematical vocabulary • Deliberate practice • Collaborative learning
2	**Specialised language of mathematics and statistics**	1. The activity looks a four strategies for vocabulary development 1.1. TIP (Term, Information, Picture) charts 1.2. Definition activity 1.3. Grids 1.4. Frayer model	• Developing mathematical vocabulary • Explicit teaching
3	**Mathematics investigation cycle**	1. Introduction to the mathematics investigation cycle 2. Doing maths using the MPTC cycle	• Explicit teaching • Fostering engagement • Scaffolding
4	**Operations on number** (Mathematical language routines)	1. Connect to the operations on number content area. 2. Three reads - mathematical language routine: using the three-reads MLR to solve operations on number problems.	• Deliberate practice • Developing mathematical vocabulary
5	**Mathematical relationships** (Co-operative learning)	1. Connect to the mathematical relationships content area. 2. Working on individual numeracy CAA activities from the introductory lesson using cooperative learning strategies.	• Collaborative learning • Deliberate practice

Lesson	Focus	Overview of activities	Acceleration Strategies
6	**Spatial properties and representations** (Co-operative learning)	1. Connect to the spatial properties and representations content area. 2. Expert Jigsaw: working in expert groups, learning about transformations, 3D plane views and 3D nets to be the "expert"; working in home groups, sharing learning.	• Collaborative learning
7	**Location and navigation** (Mathematics investigation cycle)	1. Connect to the location and navigation content area. 2. Location and navigation numeracy CAA problems. 3. Figure It Out location activities.	• Cultural responsiveness • Deliberate practice • Scaffolding
8	**Measurement** (Co-operative learning)	1. Connect to the measurement content area. 2. Using Think-Pair-Share to work on Numeracy CAA tasks. 3. Using numbered heads together strategy to work on Numeracy CAA tasks.	• Collaborative learning • Deliberate practice • Scaffolding
9	**Statistics and data** (Statistics investigation cycle - PPDAC)	1. Connect to the statistics and data content area. 2. Working on numeracy CAA problems from the introductory lesson and 2023 numeracy CAA papers with a focus on short answer questions, outcomes 1 and 2. 3. Extended questions including outcome 3 (25-30 mins).	• Deliberate practice • Scaffolding
10	**Elements of chance** (Mathematical language routines)	1. Connect to the elements of chance content area. 2. Stronger and clearer - mathematical language routine. 3. Further elements of chance problems.	• Developing mathematical vocabulary • Deliberate practice • Scaffolding

LESSON 1	Understanding the Numeracy CAA

Learning outcomes:

By the end of the lesson, learners will…

- be familiar with the three outcomes for the unit standard 32406.
- develop awareness of the seven content areas that problems within the unit standard 32406 can be set in.

Learning activities

Time	
'Do now'/ Starter activity (10 minutes)	**Vocabulary Builder** (Part 1 of 4. In preparation for lesson 2: Specialised language of mathematics and statistics) At the beginning of the lesson, signal to students to make a note of any words that they are unsure about (throughout the lesson). During the lesson students may start to identify or mention words that they are unsure of or don't know the meaning of. Capture these thoughts and ideas using a shared electronic document, or using sticky notes, or writing on a poster, or by another method.
Input/ knowledge-building activity (45 minutes)	**1. Process outcomes (35 mins)** **Introduce the unit standard to the students using slides 2-6 (2-5 mins)** 1.1 (Slide 5) Explain that the unit standard has **three process outcomes**. These process outcomes are set within mathematical and statistical content areas. The seven mathematical and statistical content areas are listed on this slide too. 1.2 (Slide 6) The unit standard, 32406, has three outcomes. If you want to show students the actual standard 32406 Apply mathematics and statistics in a range of everyday situations. To achieve the unit standard, students need to meet all three outcomes. 1.3 (Slides 7-9) These slides give a bit more detail about what is expected for each of the three outcomes. **Comparing questions for each of the different outcomes (15mins)** 1.4 Working with slides 10-24, questions from the T2 2024 paper (can also be printed, print 2 per page) N.B. Teachers could make an electronic copy of the full paper available 1.5 Students working in small groups look at the questions for each of the outcomes, and discuss how outcome 1 and 2 questions are different to outcome 3 questions, and what difference they might notice between outcome 1 questions and outcome 2 questions. For each question, discuss what is being asked, don't work it out. 1.6 Develop some interrogative questions to ask of a question to help identify if it is outcome 1, 2 or 3. E.g., Does the question ask you to explain? Then it is most likely an outcome 3 question. **Test ideas on a new set of questions (15mins)** 1.7 Working with slides 26-46, questions from the T3 2024 paper (can also be printed, can print 2 per page) 1.8 Cut the questions up so they can be sorted into the three outcomes N.B. Teachers could make an electronic copy of the full paper available 1.9 Using the interrogative questions that they have developed in the first activity with

the T2 2024 questions, students sort the T3 2024 questions into whether they think the questions are assessing outcome 1, 2, or 3.

1.10 As a class, discuss questions that are in each of the outcomes separately.
Outcome 1 - through discussion agree on the following questions, note the outcome on each question
Q1b, Q2b, Q2e, Q3a, Q4a, Q4e, Q5a, Q5d
Outcome 2 - through discussion agree on the following questions, note the outcome on each question
Q2a, Q2d, Q3c, Q3f, Q4c, Q5b, Q5e
Outcome 3 - through discussion agree on the following questions, note the outcome on each question
Q2f, Q3e, Q4b, Q4f, Q5c, Q5f

1.11 Update the interrogative questions if required

2. Content areas (10 mins)

Introduce the seven content areas to the students using slides 47-55. (5 mins)

2.1 (Slide 47) Each of the questions we have looked at in what we have just done focused on one of the process outcomes. Each question is set within the content ideas, that is, the question asks about… [next slide]
(Slide 48) one of these seven content ideas
(Slides 49-55) Here is more detail on the seven content areas or ideas [go through the slides]

N.B.: Incorporating contextually relevant examples for each of the seven content areas, so students can relate to the content with meaningful contexts, would support student engagement and learning.

Match the question to the content areas (5 mins)

2.2 Give groups of students the information on the seven content areas (pre-cut if possible), these will be headings for them to sort the questions.
* Students could make a two-way chart with the outcomes across the top and the content areas down the side. So the questions would be sorted by outcome and content.

2.3 Get students to cut up the T2 2024 questions, make sure they label the questions with outcome 1, 2, or 3.

2.4 Ask the groups of students to share the questions amongst their group. The students then look at each question they have and decide what is the main content area that the question is based on. If there is more than one, choose the main one. If they are unsure, discuss with the rest of their group.

2.5 As a class, students should discuss and agree which questions are in each of the content areas. They should be able to explain their thinking and justify their reasons (supporting Outcome 3).

See the table below for one possible way, (teachers may call some differently):

Content idea	T2 2024 Questions	T3 2024 Questions
Operations on number	3d, 3e, 4c	2a, 2b, 3c, 3f, 4a, 4b, 4e, 5b, 5d, 5f
Mathematical relationships	1c, 2a	4c
Spatial properties and representations	4f	2d, 5e
Location and navigation		
Measurement	1a, 1b, 4e	2e, 3a, 5c
Statistics and data	5f	1b, 2f, 5a
Elements of chance	4d	3e, 4f

Collect question materials in. Students will have an opportunity to revisit these in other sessions.

Reflection activity (5 minutes)

Vocabulary Builder (Part 2 of 4. In preparation for lesson 2: Specialised language of mathematics and statistics)
Check in with students that all the ideas about words that they are unsure of or don't know the meaning of are captured by the recording process decided. These will form the basis of the next lesson

Resources

Activity resources

- Targeted Support for Numeracy Intervention Planning - Supplementary PowerPoint
- Numeracy CAA T2 2024 examples. From Supplementary PowerPoint, slides12-16; 18-21, 23-24 - print single sided 2 per page
- Numeracy CAA T3 2024 examples. From Supplementary PowerPoint, slides 26-46 - print single sided 2 per page
- T2 2024 Numeracy assessment event 1 2024 paper.pdf | Schedule
- T3 2024 Numeracy assessment event 2 2024 paper.pdf | Schedule
- Content areas cards

NCEA Education direct links

- Numeracy - Learning | NCEA
- Numeracy Learning Matrix
- Unpacking Numeracy
- NCEA Co-requisite Learning Outcomes for Numeracy
- Numeracy - Teaching | NCEA
- Numeracy - Assessment | NCEA

NZQA

- 32406 Apply mathematics and statistics in a range of everyday situations
- US 32406 Assessment resources - NZQA (has assessments, schedules and reports)

Direct links to current assessments and schedules

- T2 2024 Numeracy assessment event 1 2024 paper.pdf | Schedule
- T3 2024 Numeracy assessment event 2 2024 paper.pdf | Schedule
- T2 2023 Numeracy assessment event 1 2023 paper.pdf | Schedule

- T4 2023 Numeracy assessment event 2 2023 paper.pdf | Schedule
- T2 2022 Numeracy assessment event 1 2022 paper.pdf | Schedule
- T4 2022 Numeracy assessment event 2 2022 paper.pdf | Schedule

LESSON 2	Specialised Language of mathematics and statistics

Learning outcomes:

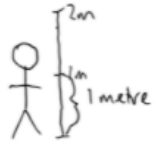
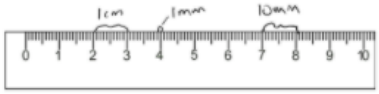
In this lesson, learners…

- Develop understanding of specialised words in mathematics and statistics
- Start to develop their own language tools to help them with vocab development.

Learning activities

Time	
'Do now'/ Starter activity (10 minutes)	**Vocabulary Builder** (Part 3 of 4. From lesson 1) Select one (or more) of the strategies below to support students to clarify the meaning and usage of different words as identified in the previous lesson. Students need to copy/ prepare to complete a chart/ table/ grid into their workbooks. **TIP (Term, Information, Picture) chart** A *TIP* chart includes mathematical vocabulary, the *TERM*, a description or *INFORMATION* about the term and *PICTURES* or diagrams to show the term. Similar to the ideas in the definition activity, except that the TIP chart is a class resource that is on the wall and as new words are introduced they are added to the TIP chart. A TIP chart becomes a point of reference for students as they learn to use the new vocabulary. **How to use:** • Create a wall chart with three columns. • Label the columns: Term, Information, Picture (TIP) • As new vocabulary is introduced, or old vocabulary is revised, add the term to the TIP chart. • Once the term has been defined the class comes up with a picture to help with understanding the term. • The teacher can provide the information (definition) of the term. N.B. TIP charts can be used for any topic in mathematics. Remember these are vocabulary tools that should be on the wall and in sight all the time for students to refer to. TIP charts support students in learning new vocabulary.

TIP Chart: Mathematics Vocabulary

Term	Information	Picture
metre (*m*)	A metre is the standard metric unit for length.	
centimetre (*cm*)	A centimetre is a unit of length. There are 100 cm in 1 m.	
millimetre (*mm*)	A millimetre is a unit of length. There are 10 mm in 1 cm. There are 1000 mm in 1 m.	

Definition activity

Students predict definitions and then follow up work enables them to revise or confirm their definitions.

How to use:

- Prepare a list of key words for the topic.
- Students predict - write down - the definitions for each of the words.
- Students undertake activities where the words are used in context.
- Students revise their definitions and confirm with given definitions.
- Follow up activities could include the student using the word in a sentence or including in their own mathematics glossary.
- Words from definition activities can be added to class TIP charts.

Students can draw up a table in their book similar to this and use it for the definition activity.

Word	Own definition	Revised definition
length		
area		
volume		
capacity		
weight		
mass		
temperature		
time		

Grids

Grids consist of a list of words down one side of the page, with another list (e.g., features) across the top. Through discussion, students mark the features that are correct for each word. Grids give students the opportunity to show and expand their understanding of known words and concepts. It also helps them to distinguish between shades of meaning.

Teachers can develop grids over time that they can come back to for identified words and concepts.

Properties	Kite	Parallelogram	Rhombus	Rectangle	Square	Isosceles trapezium
Opposite sides parallel						
Opposite sides equal						
All sides equal						
Opposite angles equal						
All angles equal						
Adjacent angles add to 180°						
Diagonals equal						
Diagonals bisect each other						
Diagonals at right angles						
Has one axis of symmetry						
Has two axes of symmetry						
Has four axes of symmetry						
Has no rotational symmetry						
Has rotational symmetry order 2						
Has rotational symmetry order 4						

Frayer model

The Frayer Model is a graphic organiser used to help students understand and learn new vocabulary or concepts. It involves defining a word or concept, describing its essential characteristics, providing examples, and noting non-examples. Here's how it typically looks:

1. **Definition:** A clear and concise explanation of the term or concept.
2. **Facts/Characteristics:** Key attributes or features that help to understand the term.
3. **Examples:** Instances or situations where the term applies.
4. **Non-examples:** Instances or situations where the term does not apply.

This model helps students deepen their understanding by encouraging them to think about what a term means, how it is used, and how it is not used. It's a great tool for building vocabulary and conceptual knowledge in a structured way.

Definition: The continuous line forming the boundary of a closed geometric figure.	**Facts/Characteristics:** Measured in units of length (e.g., metres, centimetres). Sum of all the sides of a polygon. Used to determine the boundary length of shapes.
Perimeter	
Examples: The perimeter of a rectangle with sides 5m and 3m is (5 + 3 + 5 + 3 = 16) metres. The perimeter of a square with each side 4m is (4 + 4 + 4 + 4 = 16) metres.	**Non-examples:** The area of a rectangle. The volume of a cube.

Definition: Having balanced proportions; being the same on both sides of a dividing line.	**Facts/Characteristics:** Mirror image on either side of a central line. Balanced and proportionate. Can be found in nature, art, and design.
Symmetrical	
Examples: A butterfly's wings. A perfectly round circle.	**Non-examples:** An irregular polygon. A lopsided shape.

Definition: A three-dimensional shape with six faces, all of which are rectangles	**Facts/Characteristics:** Has length, width, and height. Opposite faces are equal. Volume is calculated as length × width × height.
Rectangular prism	
Examples: A shoebox. A brick.	**Non-examples:** A sphere. A pyramid.

Input/ knowledge-building activity (40 minutes)	**Vocabulary Builder** (Part 4 of 4. From lesson 1) Complete the selected Vocabulary Builder activity (i.e. TIP chart, Definition activity table, Grids, Frayer Model). **N.B.:** For new vocabulary/ terminology/ and concepts, connect with specific content area(s).
Reflection activity (10 minutes)	**THREE, TWO, ONE Reflection activity/ Exit card** On a sticky note, students write the following reflections and stick these to the whiteboard/ hand them in as they leave: **THREE –** What are THREE new words you learnt today? **TWO –** What are TWO words you might have already known, but have learnt new meanings for today? **ONE –** What is ONE word you are still unsure about?

Resources

Activity resources

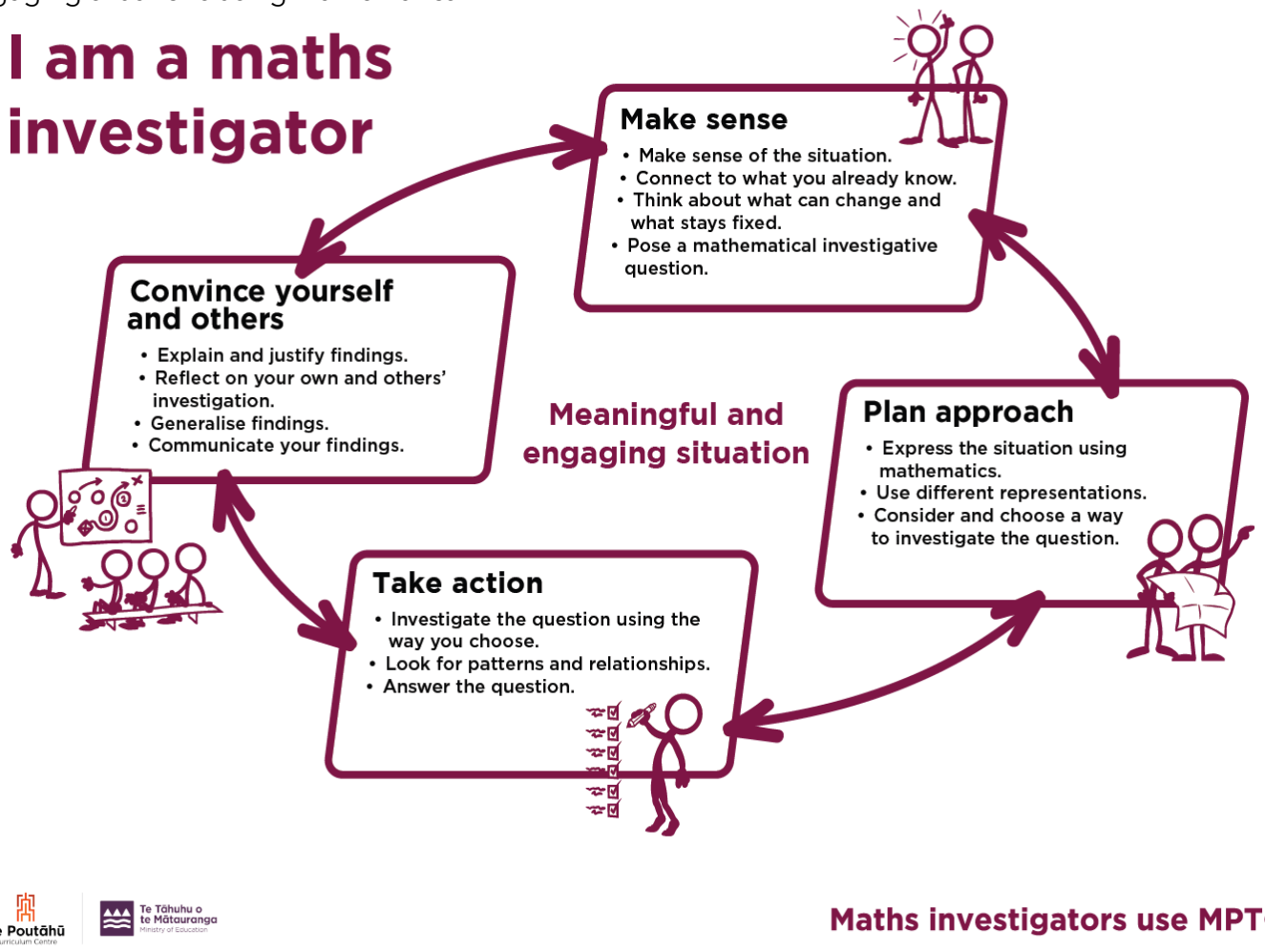
- Targeted Support for Numeracy Intervention Planning - Supplementary PowerPoint

- 2: Helping students develop the specialised language of mathematics or PDF
- NZC: Glossary of mathematics terms
- Glossary / Statistics / Home - Senior Secondary
- Frayer Model | AdLit

LESSON 3	Mathematics investigation cycle

Background information for teachers

The mathematics investigation cycle - **Make sense, Plan approach, Take action, Convince yourself and others** (MPTC) - supports students and teachers to act and identify as mathematicians. Like the statistical investigation cycle (PPDAC), users can move backwards and forwards to investigate meaningful and engaging situations using mathematics.

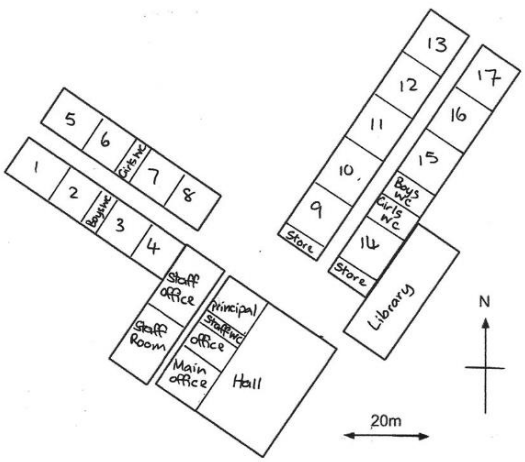


For all students the mathematics investigation cycle provides a structure to think and act like a mathematician and provides a good framework to support students who are engaging in any mathematics investigations and problem solving, but in this case specifically the numeracy CAA. The three outcomes for the numeracy CAA are connected to the mathematics investigation cycle, and the statistical investigation cycle. For example, outcome 3, which requires students to explain the reasonableness of mathematical [and statistical] responses to situations, aligns strongly with **convince yourself and others** in the mathematics investigation cycle.

Learning outcomes:

In this lesson, learners…

- are (re-)introduced to the mathematics investigation cycle and make connections to the numeracy standard.
- engage in rich learning activities using the mathematics investigation cycle.

Learning activities	
Time	
'Do now'/ Starter activity (10 minutes)	**Introduction to the mathematics investigation cycle** Introduce the students to the **mathematics investigation cycle** (or remind them of it) – (slide 57). Talk briefly about the four steps or phases: make sense, plan approach, take action and convince yourself and others. Connect to the numeracy standard outcomes, asking students to discuss in pairs ideas from the numeracy standard and where they might be visible in the mathematics investigation cycle. (They can use slide 58 to help develop their ideas).
Input/ knowledge-building activity (45 minutes)	**Doing maths using the MPTC cycle (45 mins ~ 15 mins per problem)** In this session teachers can allow students choice of the problems they do, or they could specify which one they do. Three problems are set up ready for use in this lesson plan, but the teacher could choose other problems to use from the wide selection that is available. **N.B.** the information given in this lesson plan is the same as is on Tāhūrangi. It is collated here for your convenience. See Level 4 rich learning activities | Level 5 rich learning activities. Activities in this lesson plan • Problem 1 | Where's the wifi (location and navigation focus) • Problem 2 | Andrew's caravan (measurement focus) • Problem 3 | Tennis costs (mathematical relationship focus) **Problem 1 | Where's the wifi** • Where's the wifi? • Student activity sheet Hand out or share the problem with students. This could be done electronically by giving them the link to the student activity sheet copy of the problem. **The problem** Arthur, Betty, and Charlotte are all at school, in different rooms, successfully using their friend Darren's WiFi hotspot from his phone. Arthur and Betty can connect within 50 m of Darren's phone, but Charlotte's phone can manage 80 m. The map of the school shows where Arthur, Betty and Charlotte are at the moment. Darren is due east of Arthur who is in room 3. Betty is in room 13 and Charlotte, in room 11 is northeast of Darren. Where is Darren? 

Using the mathematics investigation cycle

In the activity on Tāhūrangi prompts are given for each of the four phases of the mathematics investigation cycle. The prompts are listed below with notes for teachers, they are also on the Supplementary PowerPoint (slides 59-73) without the notes for teachers.

Make sense

Introduce the problem. Allow students time to read it and discuss it in pairs or small groups.

- Do I understand the situation and the words? (Students may need support to understand the context of a WiFi hotspot, and the range of that WiFi.)
- What is this problem about? What am I being asked to find out?
- What is the important information? (The conditions of how far Arthur, Betty and Charlotte are from Darren's phone and the direction Darren is from Arthur.) What information is known? (Arthur is in room 3.)
- Where else in my life/the world can I see this happen? (Students may have solved problems involving direction and position before.)
- What will my solution look like? (The solution will be the location of Darren on the map with justification for that location.)

Plan approach

Discuss ideas about how to solve the problem. Emphasise that, in the planning phase, you want students to say how they would solve the problem, not to actually solve it.

- What would be a sensible first strategy? What information should I work with first?
- What are the maths skills I need to work this out?
- How could I show this problem using the map?
- Do I anticipate how I will know where Darren is? (Students should consider that there will be a convergence of possible area from processing all the clues.)
- What tools (digital or physical) could help my investigation? (Use of a drawing compass to create arcs of possible locations, from the distance clues, will be important.)

Take action

Allow students time to work through their strategy and find a solution to the problem.

- Have I shown my workings in a step-by-step way by recording on the map?
- How will I systematically account for all the information that I have?
- Is there another possible answer or way to solve it? Would the order of using the information make a difference to the result?
- Is my strategy looking fruitful, or do I need to try something else?
- How do my results look different to others? Why could this be?
- Does my solution make sense? Does it match all the clues?
- Does my solution answer the question? Have I specified Darren's location precisely?

Convince yourself and others

Allow students time to check their answers and then either have them pair share with other groups or ask for volunteers to share their solution with the class.

- What is the solution?
- Does my solution match all the given information?
- Is my working clear for someone else to follow?

- How would I convince someone else I am correct?
- Could I have solved the problem in a more efficient way?
- What connections can I see to other situations? What situations would my ideas and strategies work for?
- Is there some mathematics that I need to learn?

Solutions

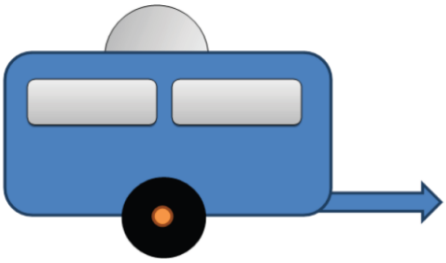
Example solutions are given in the task online. See examples of work.

Problem 2 | Andrew's caravan

- Andrew's caravan
- Student activity sheet

Hand out or share the problem with students. This could be done electronically by giving them the link to the student activity sheet copy of the problem.

The problem

Andrew wants to store his caravan in a shed with a clearance of 2.25 m to get into the shed. The caravan body is 1.85 m high and is on wheels with a diameter of 40 cm. He is going to fit a dome ceiling window in the roof of the caravan. What is the maximum height the dome could be so that he can still use the shed for storage?	

Using the mathematics investigation cycle

In the activity on Tāhūrangi prompts are given for each of the four phases of the mathematics investigation cycle. The prompts are listed below with notes for teachers, they are also on the Supplementary PowerPoint (slides 59-73) without the notes for teachers.

Make sense

Introduce the problem. Allow students time to read it and discuss it in pairs or small groups.

- Do I understand the situation and the words? (Students may need support to understand the meaning of diameter, caravan body, and dome.)
- What is this problem about? What am I being asked to find out?
- Can you visualise the measurements on the picture of the caravan? Estimate how high the shed doorway is.
- Where else in my life/the world can I see this happen? (Students may have solved problems involving direction and position before.)
- What will my solution look like? (The solution will be the height of the dome, so the caravan can still fit into the shed.)
- Do I have all the information I need to get started?

Plan approach

Discuss ideas about how to solve the problem. Emphasise that, in the planning phase, you want students to say how they would solve the problem, not to actually solve it.

- What else do I need to know to get started?
- How will I record my working so I can cope with all the information?
- Can I express all the measurements in the same unit? Why will that help?
- What tools (digital or physical) could help my investigation? Will using a ruler or tape measure help me understand the real sizes of the caravan and shed?

Take action

Allow students time to work through their strategy and find a solution to the problem.

- Have I shown my workings in a clear way using a diagram?
- Do my calculations seem correct? Do they match any estimates I made?
- Does my solution seem reasonable?
- Is there another possible way to solve this problem?

Convince yourself and others

Allow students time to check their answers, and then either have them pair share with other groups or ask for volunteers to share their solution with the class.

- Are my workings clear for someone else to follow?
- How would I convince someone else that I am correct?
- How might Andrew use the advice my solution gives him?
- Would my answer work in a different situation?
- What have I learned about using units of length?
- Which ideas or tools worked well in my investigation?

Solutions

Example solutions are given in the task online. See examples of work.

Problem 3 | Tennis costs

- Tennis costs
- Student activity sheet

Hand out or share the problem with students. This could be done electronically by giving them the link to the student activity sheet copy of the problem.

The problem

Charlie plays tennis, with one ladder competition day each week.

It costs $70 for the season's subs, and his mother drives him 10 km each way to the tennis club (running costs for the car are 70 cents per km).

- If the season runs for 14 weeks, what is the cost of competing in this tennis club ladder?
- If he finds a way to share transport with another player (so his mother only needs to take him every second week), what would the cost for the season become?
- Are the savings worthwhile?

Using the mathematics investigation cycle

In the activity on Tāhūrangi prompts are given for each of the four phases of the mathematics investigation cycle. The prompts are listed below with notes for teachers, they are also on the Supplementary PowerPoint (slides 59-73) without the notes for teachers.

Make sense

Introduce the problem. Allow students time to read it and discuss it in pairs or small groups.

- Do I understand the situation? (Students may need background information about finding total costs by multiplying with unit rates.)
- What is this problem about? What am I being asked to find out?
- What is the cost? How do you work them out? Which costs are fixed, and what costs vary?
- What information has been given, and what more information might be needed?
- Can I anticipate what the total cost will be? Can I anticipate the cost difference if transport is shared?
- Does this look or sound like a problem I have worked on before? What kind of problem is this?
- Where else in my life/the world can I see this happen? (Students may have solved problems involving direction and position before.)
- What will the answer look like? (A total cost for the 14 weeks and a cost with shared transport factored in. The costs are supported by clear working.)

Plan approach

Discuss ideas about how to solve the problem. Emphasise that, in the planning phase, you want students to say how they would solve the problem, not to actually solve it.

- What are the maths skills I need to work this out? (Rates are integral to the problem.)
- How could I show this problem using numbers, equations, pictures, graphs, tables, or materials? (A ratio table is a good strategy to use.)
- What strategies can I use to get started?

	• How much change in the total cost is shared transport likely to cause? How do you know? Can I make an estimate? • What tools (digital or physical) could help my investigation? **Take action** Allow students time to work through their strategy and find a solution to the problem. • Have I shown my working in a clear, systematic way? • Does my answer seem correct? Is it close to my estimation? • How could I make sure that I haven't missed anything? • Do the costs seem reasonable? Is sharing transport worthwhile? Is there a downside to sharing transport? • Do my results look the same or different from others? Why could this be? • Is there another possible answer or way to solve the problem that is more efficient? **Convince yourself and others** Allow students time to check their answers, and then either have them pair share with other groups or ask for volunteers to share their solution with the class. • What is the cost difference between full and shared transport? • Have I shown clearly how the costs were found? • How would I convince someone else that my calculations are correct? • Is there some mathematics that I need to learn? • What does my answer mean for people who want to share transport costs? **Solutions** Example solutions are given in the task online. See examples of work.
Reflection activity (5 minutes)	**Check in at the end** At the end of the lesson check in with the students, reconnecting the MPTC cycle with the three outcomes for the numeracy standard. Get students to reflect on their learning from the lesson, using the intended learning outcomes.

Resources

Activity resources

Targeted Support for Numeracy Intervention Planning - Supplementary PowerPoint

Published MPTC materials on Tāhūrangi

- The Mathematics Investigation Cycle – poster and cards
- Poster
- Cards

Rich learning activities - set up with MPTC structure

- Level 4 rich learning activities
- Level 5 rich learning activities

Specific rich learning activities used in this lesson plan

- Where's the wifi? | Student activity sheet
- Andrew's caravan | Student activity sheet
- Tennis costs | Student activity sheet

- Mathematics investigation cycle (from Supplementary PowerPoint, slides 56-73)

Student resources for the session

- Maths Inv Cycle and Numeracy Standard (from Supplementary PowerPoint, slides 58, print for pairs or groups A3)

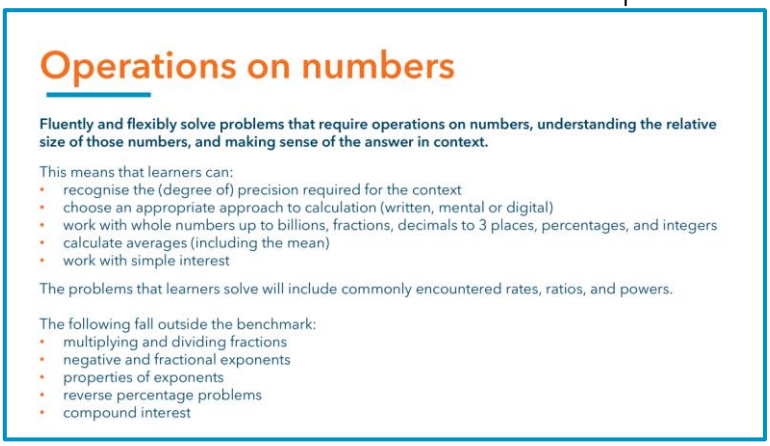
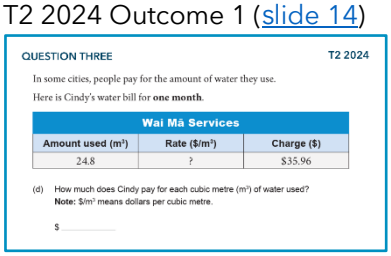
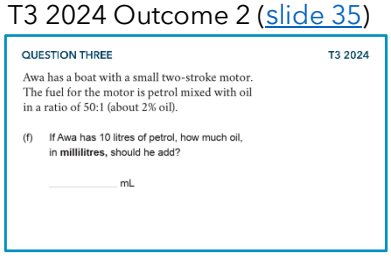
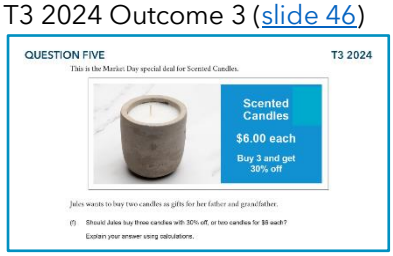
LESSON 4	**Operations on number** (Mathematical language routines)

Learning outcomes:

In this lesson, learners…

- are solving numeracy problems with a focus on *operations on number*.
- are introduced to the mathematical language routine of *three reads* to support them to read and unpack what is in a problem so they can work on answering the question.
- have accelerated support needed with a number focus is identified.

Learning activities

Time	
'Do now'/ Starter activity (5 minutes)	**Introduction and scene setting** Remind students about the content ideas for operations on number (slide 49).  **Operations on numbers** **Fluently and flexibly solve problems that require operations on numbers, understanding the relative size of those numbers, and making sense of the answer in context.** This means that learners can: • recognise the (degree of) precision required for the context • choose an appropriate approach to calculation (written, mental or digital) • work with whole numbers up to billions, fractions, decimals to 3 places, percentages, and integers • calculate averages (including the mean) • work with simple interest The problems that learners solve will include commonly encountered rates, ratios, and powers. The following fall outside the benchmark: • multiplying and dividing fractions • negative and fractional exponents • properties of exponents • reverse percentage problems • compound interest
Input/ knowledge-building activity (45 minutes. Approx. 15 minutes per problem)	**Three reads - mathematical language routine** Initially students will explore three problems from the 2024 numeracy papers. All three problems require **operations on number**. They will be introduced to the mathematical language routine (MLR) of **three reads** to help them to understand the problem and start to think about ways to solve it. The three reads MLR can also support the **Make sense** and **Plan approach** phases of the **mathematics investigation cycle**. Students are working in small groups. 1. The three problems used are from the session (1) on **Understanding the Numeracy CAA**, and include an example from Outcome 1, Outcome 2 and Outcome 3. The problems are: T2 2024 Outcome 1 (slide 14)  QUESTION THREE T2 2024 In some cities, people pay for the amount of water they use. Here is Cindy's water bill for **one month**. Wai Mā Services Amount used (m^3) | Rate ($/m^3$) | Charge ($) 24.8 | ? | $35.96 (d) How much does Cindy pay for each cubic metre (m^3) of water used? **Note:** $/$m^3$ means dollars per cubic metre. $ _____ T3 2024 Outcome 2 (slide 35)  QUESTION THREE T3 2024 Awa has a boat with a small two-stroke motor. The fuel for the motor is petrol mixed with oil in a ratio of 50:1 (about 2% oil). (f) If Awa has 10 litres of petrol, how much oil, in **millilitres**, should he add? _____ mL T3 2024 Outcome 3 (slide 46)  QUESTION FIVE T3 2024 This is the Market Day special deal for Scented Candles. Scented Candles $6.00 each Buy 3 and get 30% off Jules wants to buy two candles as gifts for her father and grandfather. (f) Should Jules buy three candles with 30% off, or two candles for $6 each? Explain your answer using calculations. 2. Introduce the activity to the students explaining that we are going to use a strategy called **three reads** to understand the problem and to help us to solve it.

- Three reads is one of the eight mathematical language routines (Zwiers et al., 2017, Amplify Desmos Maths, 2025)
- Each of the three problems is set up in the supplementary PowerPoint (slides 74-86) for the three reads.
  - The first read, block numbers and the question (first slide)
    - Ask what is this about? What is the context of the situation? Describe in your own words.
  - The second read, block the question (second slide)
    - Ask what are the quantities in the problem? What can be counted or measured? Identify numbers and units in context.
  - The third read, brainstorm possible mathematical solution methods (third slide)
    - How might you go about solving this? Take time to think about how you could get started, what representation might help you… don't solve it yet.
- As a group work together to answer the problem, sharing approaches and solutions across the class.

In this next activity students use the three reads MLR on problems without the scaffolding provided in previous problems, i.e., without the numbers and the question being blocked out.

1. Discuss with students how they might do this for themselves.
   - They might suggest covering the question with their hand and then
     - For the first read, just focus on the context and make a note about it
     - For the second read, now consider the numbers, maybe underlining them or highlighting them
     - For the third read, uncover the question.

2. Provide students with the handout so they can continue to use the three reads MLR to solve the problems. The handout starts with problems with some scaffolding and moves to questions with no scaffolding.

References for solutions
From the supplementary PowerPoint slides
T2 2024 Q3d
T3 2024 Q3f | Q5f

From the handout
T4 2023 Q1f | Q5b
T2 2023 Q1a | Q1b | Q1c | Q2c | Q5d
T2 2022 Q4a | Q4b | Q4c | Q8a

Additional problems if required
Refer back to the materials from the first session understanding the numeracy CAA. Three of the problems have been done when learning about the three reads initially. These are crossed out in the table. The remaining questions can be used if additional problems are required.

Content idea	T2 2024 Questions	T3 2024 Questions
Operations on number	~~3d,~~ 3e, 4c Schedule	2a, 2b, 3c, ~~3f,~~ 4a, 4b, 4e, 5b, 5d, ~~5f~~ Schedule

Reflection activity (10 minutes)

Check in at the end

At the end of the lesson check in with the students on the operations on number content area ideas from the start (Slide 46), connecting the activities with the different operations on number content areas.

Get students to reflect on their learning from the lesson, using the intended learning outcomes.

Resources

Activity resources

- Targeted Support for Numeracy Intervention Planning - Supplementary PowerPoint
- Operations on number session - additional problems
- US 32406 Assessment resources - NZQA examples are sourced from 2022-2024 previous numeracy CAA tasks
- T2 2024 Numeracy assessment event 1 2024 paper.pdf | Schedule
- T3 2024 Numeracy assessment event 2 2024 paper.pdf | Schedule
- T2 2023 Numeracy assessment event 1 2023 paper.pdf | Schedule
- T4 2023 Numeracy assessment event 2 2023 paper.pdf | Schedule
- T2 2022 Numeracy assessment event 1 2022 paper.pdf | Schedule
- T4 2022 Numeracy assessment event 2 2022 paper.pdf | Schedule

Additional supports

Accelerated support for number

During this session and from prior knowledge of your students they may have some identified gaps. One option is to run short 10-15 minute sessions to support students with some of these content ideas. Using the supports available on Tāhūrangi is one option for teachers. The supports that relate to number are listed below with direct links to the Tāhūrangi activity.

- Fractions in ratios
  - Comparing ratios
  - Identifying fractions in simple ratios
  - Equivalent fractions in simple ratios
  - Comparing ratios with a common whole
  - Comparing ratios
- Putting decimal place value to work
  - Ordering decimals
  - Locating decimals on a scale
  - Adding and subtracting decimals
  - Multiplying decimals by 10 or 100

- Percentages as change (% can be more than 100%)
  - Simple discounts
  - Comparing in both directions
  - Percentage increases
  - More complex percentage increases
  - More complex discounts

- Using percentages to represent part-whole relationships
  - Simple fractions as percentages
  - More complex fractions as percentages
  - Scaling up to 100%
  - Rounding to estimate percentages
  - Finding the missing part
  - Finding the whole

- Rates – using multiplication and division
  - Unit rates
  - Applying multiplication and division to rates
  - Working with rates and fractions
  - Working with equivalent rates
  - Finding unit rates

References

Amplify Desmos Maths, 2025. *Asset-Based Assessment: What it is and why it's needed in K-12 classrooms.*

Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the design of mathematics curricula: Promoting language and content development. *Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources* .

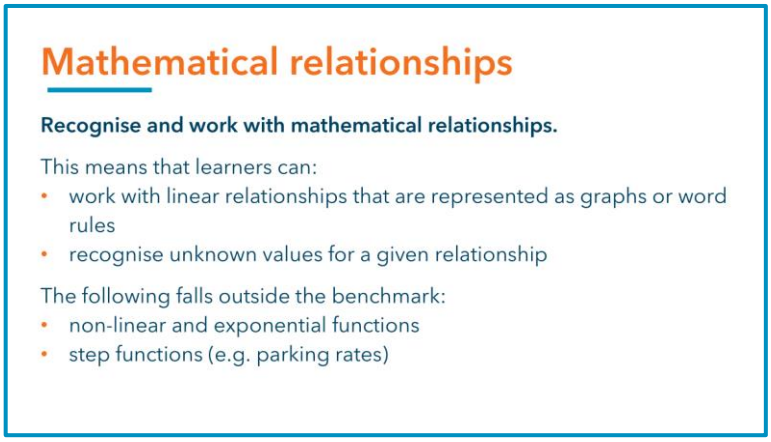
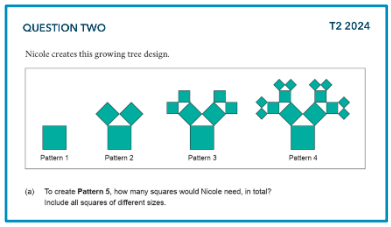
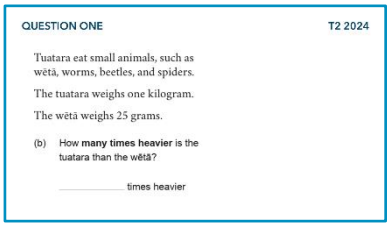
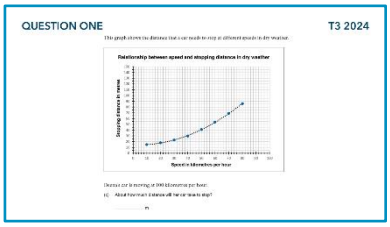
LESSON 5	**Mathematical relationships** (Co-operative learning)

Learning outcomes:

In this lesson, learners…

- are solving problems with a focus on *mathematical relationships*.
- are using cooperative learning approaches to solve these problems.

Learning activities

Time	
'Do now'/ Starter activity (10 minutes)	**Introduction and scene setting** Remind students about the content ideas for mathematical relationships (slide 50).  **Mathematical relationships** **Recognise and work with mathematical relationships.** This means that learners can: • work with linear relationships that are represented as graphs or word rules • recognise unknown values for a given relationship The following falls outside the benchmark: • non-linear and exponential functions • step functions (e.g. parking rates) Discuss the **THINK-PAIR-SHARE** strategy, introducing or reminding students about it.
Input/ knowledge-building activity (40 minutes)	**1. Using Think-Pair-Share to work on Numeracy CAA tasks (20-30 mins)** In this initial activity students are going to work through three of the Numeracy CAA tasks that were looked at during the first session. Select the three mathematical relationship questions, T2 2024 Q2a, T2 2024 Q1c and T3 2024 Q4c. Or use this document which has all three problems on it. These problems are also on the Numeracy CAA Google slides as outlined below. The problems are: T2 2024 Outcome 1(slide 13) T2 2024 Outcome 2 (slide 18) T3 2024 Outcome 2 (slide 38)  QUESTION TWO T2 2024 Nicole creates this growing tree design. Pattern 1 Pattern 2 Pattern 3 Pattern 4 (a) To create Pattern 5, how many squares would Nicole need, in total? Include all squares of different sizes.  QUESTION ONE T2 2024 Tuatara eat small animals, such as wētā, worms, beetles, and spiders. The tuatara weighs one kilogram. The wētā weighs 25 grams. (b) How **many times heavier** is the tuatara than the wētā? _____ times heavier  QUESTION ONE T3 2024 Relationship between speed and stopping distance in dry weather Working on one problem at a time - repeat for all three problems. 1.1 Students try the problem, they might like to use the **three reads** MLR from the previous session to make notes about the problem and how they could solve it and if they can, they solve the problem. 1.2 Students pair up and both students share their ideas about how they solved the problem, each student explaining their method and solution, even if they are the

	same; or if they haven't solved it, what ideas did they have to try to solve it. Work together to find a solution. 1.3 Pairs are paired. One person from each pair shares their solution and methods with the other pair. The person who shares changes each time they pair up. **2. Using numbered heads together strategy to work on Numeracy CAA tasks (20-30 mins)** For these next activities rather than students thinking by themselves, then sharing, they are working from the start as a group on the problem using the **numbered heads together** strategy. Students may choose to work individually before coming to share their ideas, but this is not an expectation. 2.1 Explain to the students that they will work together to find the solution to the problems given to them. The students in a group are numbered and then the teacher calls a number, and the student with that number shares the group's answer. This means that everyone in the group needs to understand the group's response. There are two complete questions that students can work on using the numbered heads together strategy. • Mathematical relationships T2 2022 Q7.pdf | This question has three parts: ○ (a) Outcome 1, operations on number ○ (b) Outcome 1, operations on number ○ (c) Outcome 3, mathematical relationships ○ Schedule • Mathematical relationships T4 2022 Q1.pdf | This question has three parts: ○ (a) Outcome 1, operations on number ○ (b) Outcome 2, operations on number ○ (c) Outcome 3, mathematical relationships ○ Schedule 2.2 Share one question at a time. **N.B.:** Students collaborating, discussing and explaining their thinking (to the group) is an important part of the learning process – ensure there is sufficient time for this to happen. 1. Members of the group work together to find the solutions to all parts of the question. 2. Allow time to solve the problem and then to make sure everyone in the group understands the solution. 3. Remind students that you will pick a number and that numbered person speaks on behalf of the group.
Reflection activity (10 minutes)	**Check in at the end** At the end of the lesson check in with the students on the mathematical relationships content area ideas from the start (slide 50), connecting the activities with the different mathematical relationships content areas. Get students to reflect on their learning from the lesson, using the intended learning outcomes.

Resources

Activity resources

- Targeted Support for Numeracy Intervention Planning - Supplementary PowerPoint

Cooperative learning resources

- S4. Cooperative learning (think-pair-share & numbered heads together)

Additional supports

Should you require them, here are some additional supports.

- In noticing and acting links are given to existing accelerating learning activities on Tāhūrangi.
- The Figure it out resources also provide additional teaching and learning activities on mathematical relationships. These activities are in the style of the numeracy standard activities.

Noticing and acting

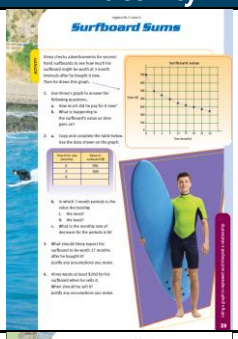
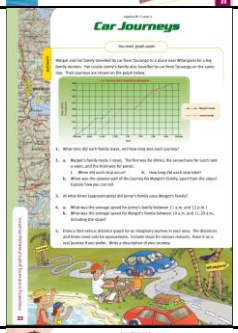
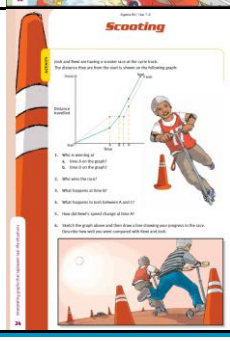
Notice where students might have difficulties with questions and identify if any of the accelerating learning activities might be useful to support knowledge building.

There is one set of algebra activities in the accelerating learning materials. The main page has a set of diagnostic questions that can be used to identify student needs.

- Finding and expressing relationships (main page)
  - Reasoning from one shape
    - The purpose of this activity is to support students to find structure in one shape within a growing pattern. Relating parts of the single shape to the shape number offers possibilities for creating function rules.
  - Graphing relations
    - The purpose of this activity is to support students to represent linear relations on a number plane.
  - Expressing general rules
    - The purpose of this activity is to support students to express general (function) rules using algebraic notation. To avoid unnecessary "noise", the students work only with tables of values.
  - Working backwards
    - The purpose of this activity is to support students to apply linear relationships to find unknown values of the explanatory variable, the x-value or value of n, when given the value of the response variable, the y-value or nth term. At this level, students are not expected to master solving linear equations, though the strategy should be introduced in context.
  - Creating function rules
    - The purpose of this activity is to support students to develop function rules that connect the shape number with the number of squares. Tables of values are used to make pattern spotting easier. Function rules may or may not be expressed using algebraic symbols.

Figure It Out activities

A search of Figure It Out books found these three activities that have graphs for interpreting.

Name of the activity (and link)	Description	Thumbnail of the activity
Surfboard sums FIO activity	In this activity, students interpret the graph showing how the value of material goods such as a surfboard usually decreases over time.	
Car journeys FIO activity	In this activity, students first interpret distance–time graphs that illustrate two car journeys from Tauranga to Whangarei on the same day. While distance–time graphs may seem clear and simple to interpret, many students see quite different messages depicted in the graphs.	
Scooting FIO activity	This activity asks the students to interpret line graphs illustrating a scooter race between Josh and Rewi.	

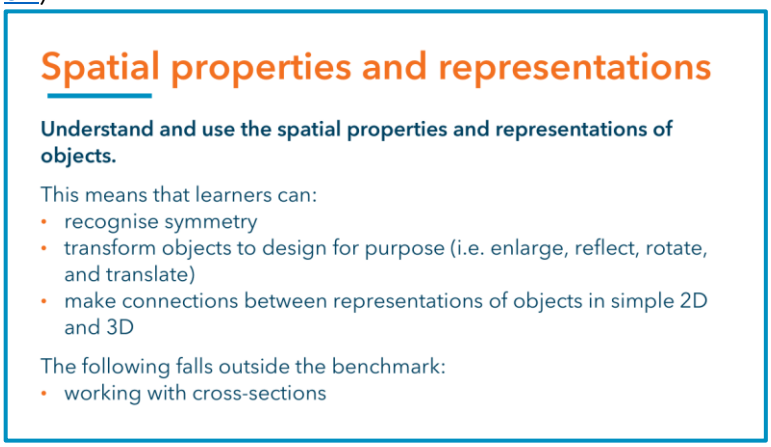
LESSON 6	**Spatial properties and representations** (Co-operative learning)

Learning outcomes:

In this lesson, learners…

- are solving problems with a focus on *spatial properties and representations*.
- are using cooperative learning approaches to solve these problems.

Learning activities

Time	
'Do now'/ Starter activity (10 minutes)	**Introduction and scene-setting** Remind students about the content ideas for spatial properties and representations (slide 51).  Introduce the cooperative learning strategy – **Expert Jigsaw**. ● Jigsaw is a cooperative learning strategy that enables each student of a "home" group to focus on one area, idea or 'piece of the puzzle'. ● Students become 'experts' in their assigned area by working with members from other groups who are also assigned that same focus. These new groups are called 'expert' groups. ● The 'experts' then return to their 'home' group and share their expertise, which is required for the completion of the home group task. As in a jigsaw puzzle, each piece or each student's part is essential to enable the completion and understanding of the home group task. In this lesson we will be working on exploring three activities. ● Working in home groups of three, each person from our home group joins an expert group to become an expert in that element. ○ Teacher note: expert groups could be 4-10 in size, depending on the size of your class. ● Once we have had our expert training, we go back to our home group with our expertise to work on the activities. Each member of the group is given one of the activities. They take their activity and move to their expert group.

Input/ knowledge-building activity (15 minutes)

Expert Jigsaw

Expert Jigsaw | Part 1: Working in expert groups (10-15 mins)

1. Move students into their expert groups.
    - There should be three expert groups with as many members as there are groups of three in the class.
    - If your numbers don't exactly divide by 3 then up to two home groups will have four students.
    - In this case two students will be numbered the same number from 1-3. You might strategically double number in this case.

2. Allocate an activity to each of the expert groups. They need to have:
    - a copy of the Figure it out activity
    - a copy of the group instructions
    - any physical or other materials required
    - Make the student write on sheets available as sometimes the expert group also writes on these as part of their becoming experts.

3. The teacher also needs to have the solutions copied ready to hand out once the expert groups have finished their work so they can check.

4. Check in frequently with the different groups to help them become the experts, providing support and guidance.

5. The timing of this section may need to be a bit longer, keep an eye on progress, but not too much longer as they still have to share back in their groups. If time allowed, e.g. a second lesson, longer time could be given for each activity.

Preparation

The table (next) outlines the three activities with a brief description of what each group does and the resources required.

Topic	Activity and notes	Resources required
Translation, reflection, rotation	Shifting shapes Expert groups work through the two questions focusing firstly on the language of transformations and then creating an image using transformations. Back in their home group they support their group to do the descriptions of the transformations and then to make the transformation.	Figure it out activity (1 per expert) Expert group notes (1 per expert) Student write on sheet (1 per student, print page 1/2, that should be enough for the time, but the second page (2/2) could be follow up work to do) Group instructions (1 per expert) Solutions (1 for expert group) Solutions for the second page of problems can be found on the task page.
3D plane views	A different view Expert groups do the two questions, creating the 3D objects using multilink cubes if possible. Back in their home group they support	Figure it out activity (page 1 only, 1 per expert) Student write on sheet (1 per student) Group instructions (1 per expert) Solutions (1 per expert group)

	their group to identify the views for the different 3D shapes. The student write on sheet only has three of the objects from Q2.	Solids if available - cone, sphere, cylinder, half-cylinder, cube, square-based pyramid (or photographs) Multilink cubes (15 per expert)
3D nets	Open and shut Expert group works with the problem using the nets to check their solutions. They discuss how they can use the visuals only to solve the problem and what questions or hints can they give. Back in their home group they support their group to solve the problem, bringing out the actual nets only as needed. **The problem:** Q1 only, working from the visuals to decide which ones make an open box and which ones don't.	Figure it out activity (1 per expert) Copymaster (1 per student, could be A5 size for home groups, but need 1 A4 per expert) Group instructions (1 per expert) Solutions (1 per expert group) Scissors (1 per expert) Teacher note: a, c, d, e, g, i, j, k can be made into open boxes.

Expert Jigsaw | Part 2: Home group to work on the problems (40-45 mins)
In organising the home group work, set them up to have about 12-15 mins on each activity/topic. If they finish early they can move onto the next activity, but they can't go over the time allocation. Give clear indications when it is time to move onto the next activity.

Reflection activity (10 minutes)

Check in at the end
At the end of the lesson check in with the students on the spatial properties and representations content area ideas from the start (slide 51), connecting the activities with the different spatial properties and representations content areas.
Get students to reflect on their learning from the lesson, using the intended learning outcomes.

Resources

Activity resources

- Targeted Support for Numeracy Intervention Planning - Supplementary PowerPoint

Teaching approaches

- S4. Cooperative learning (jigsaw)

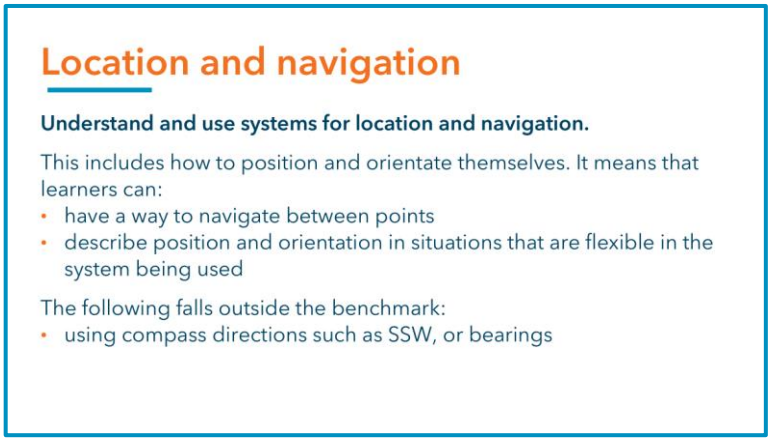
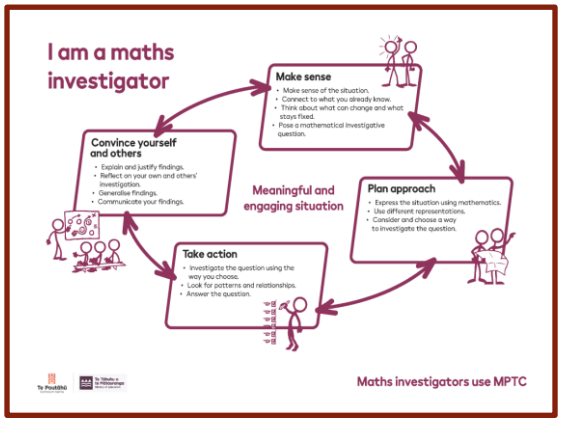
LESSON 7	**Location and navigation** (Mathematics investigation cycle)

Learning outcomes:

In this lesson, learners…

- are solving problems with a focus on *location and navigation*.
- are using the mathematics investigative cycle to solve these problems.

Learning activities

Time	
'Do now'/ Starter activity (10 minutes)	**Introduction and scene setting** Remind students about the content ideas for location and navigation (slide 52).  **Location and navigation** **Understand and use systems for location and navigation.** This includes how to position and orientate themselves. It means that learners can: • have a way to navigate between points • describe position and orientation in situations that are flexible in the system being used The following falls outside the benchmark: • using compass directions such as SSW, or bearings Touch base on the **mathematics investigation cycle** (slide 57) and the **three reads** strategy to help them focus on their solving strategies for the activities. Ask students what they remember about the maths investigation cycle and three reads MLR. Use the suggested resources to help students reactivate their knowledge about them.  I am a maths investigator Make sense Plan approach Take action Convince yourself and others Meaningful and engaging situation Maths investigators use MPTC **First read** What is this about? What is the context of the situation? Describe in your own words. **Second read** What are the quantities in the problem? Identify numbers and units in context. **Third read** How might you go about solving this? How could you get started?

	In this session we are going to work on problems about location and navigation and solve these using previous strategies we have learnt.
Input/ knowledge-building activity (40 minutes)	**Location and navigation numeracy CAA problems (30-40 mins)** There are four location and navigation problems that have been sourced from the numeracy CAA. All location and navigation problems have the maps redacted so it makes it trickier to access these problems. The four problems have been adapted for the mathematics investigation cycle. They can be found here. These can be printed for students to write on, using the mathematics investigation cycle. 1. Hand the problems out to small groups of students. • Do one problem initially together supporting students to understand the problem, using the three reads strategy. • Support students to use technology to access maps, or prepare your own maps for them to use. Copyright doesn't allow maps to be shared in this resource. Original questions from numeracy CAA

T2 2024 Q1f Outcome 2 Half of all the tuatara in New Zealand live on an island that is about 95 km northwest of Wellington. (f) Write the letter that shows the location of the island. Use the scale on the map to help you. *The map is redacted, so students will need to source their own map from Google maps.* Schedule	**T4 2023 Q1a Outcome 1** Māori sailed from places like Tahiti to settle in Aotearoa New Zealand. They came in waka hourua which are large canoes with twin hulls. (a) Select (✓) the compass direction of the trip from Tahiti to Aotearoa New Zealand: West South South-east North-west South-west *The map is redacted, so students will need to source their own map from Google maps.* Schedule
T4 2023 Q1d Outcome 1 This map shows the route taken by a vaka on a journey from Auckland to Hawai'i. (d) Using the scale on the map, which of the following estimates is closest to the total distance of the trip? • 5,000 km • 7,000 km • 9,000 km • 11,000 km • 13,000 km *The map is redacted, so students will need to source their own map from Google maps.* Schedule	**T2 2022 Q9a Outcome 1** Māori regard kūaka (bar-tailed godwits) as birds of mystery. That is because it is believed that kūaka travel through the ancestral home of Hawaiki. They migrate between New Zealand and Alaska each year as shown on the map below. Kūaka make a non-stop flight from Alaska to arrive in New Zealand in September. (a) Papua New Guinea is the second of the 'stopover sites'. It is marked by the purple spot on the map. In which direction is the flight path from New Zealand to the stopover site in Papua New Guinea. • North • West • North-east • North-west *The map is redacted, so students will need to source their own map from Google maps.*

	Schedule

2. Support students during the different phases of the mathematics investigation cycle using prompts such as those suggested below.

Make sense:

1. Has anyone seen a problem like this before?
2. What are the important ideas in this problem?
3. Can you rephrase the problem in your own words?
4. What is this problem asking you to find out?
5. What information has been given?
6. What conditions apply?
7. Can you guess what the answer might be?

Plan approach:

1. What strategies might you use to get started?
2. Which of these ideas are worth pursuing?
3. What do you need to know to get started?
4. How could you show this problem using numbers, pictures, graphs, tables or materials?
5. What maths skills do you need to work this out?
6. Could you try a simpler problem?
7. What tools do you need to help you get started?

Take action:

1. Tell me what you are doing?
2. Why (How) did you think of that?
3. Why are you doing this?
4. What will you do with the result of that work when you've got it?
5. Why is this idea better than that one?
6. You've been trying that idea for 5 minutes. Is it time to try something else?
7. Can you justify that step?

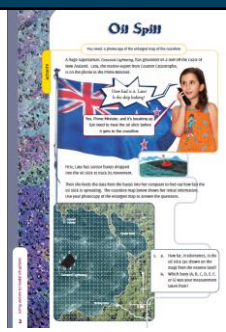
Convince yourself and others:

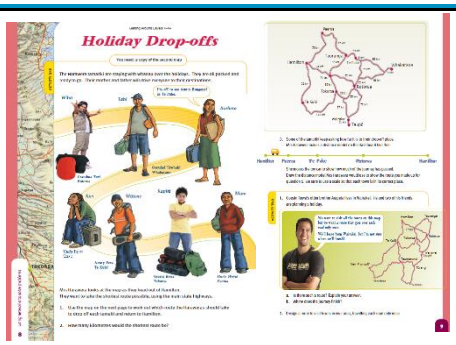
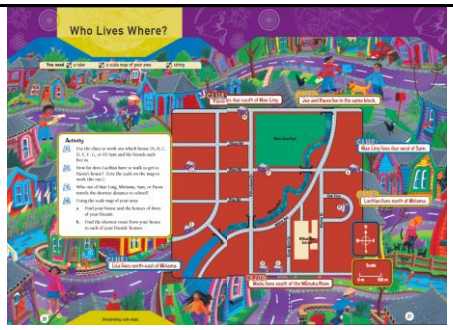
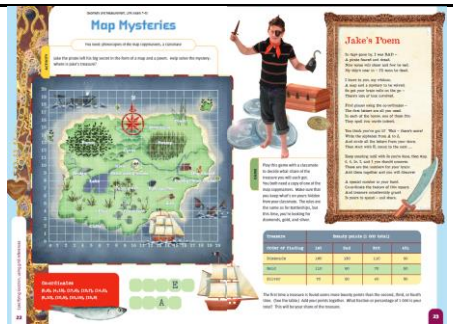
1. Have you answered the question?
2. Have you considered all possible cases?
3. Have you checked your solution?
4. Does the answer look reasonable?
5. Is there another answer?
6. Is there another solution?
7. Can you explain your solution to the class?
8. Is there another way to solve the problem?
9. Can you generalise or extend the problem?

3. Figure It Out location activities (10-20 mins)

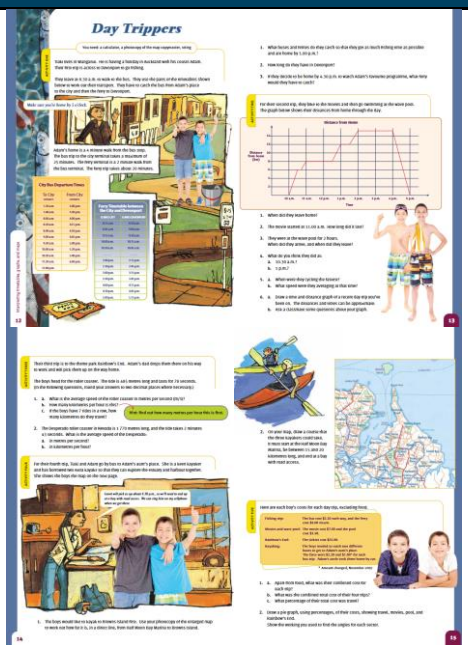
One of the challenges is to find activities that have maps that can be shared. On Tāhūrangi are all the Figure It Out book activities and many of these include location and navigation, and maps.

The number of activities you can work through will depend on the time available. The table below gives the link to the activity and some detail about what is involved in the activity. This will allow choices based on identified student needs.

Name of the activity (and link)	Description	Thumbnail of the activity
Oil spill **Q1** FIO activity Copymaster	Question 1 (page 1) requires students to use a scale to find distances. Teachers could supplement with questions about the compass direction of various places (e.g., gannet colony, oyster farm, shrimp farm) from the oil spill. *Q2-3. Draw pathways using vectors.* *Interpret points and lines on coordinate planes, including scales and bearings on maps.*	
Treasure island FIO activity Copymaster	Use compass directions and scale drawing. Communicate and interpret locations and directions, using compass directions, distances, and grid references.	
Ringing the road FIO activity Copymaster (see pages 14-15)	Apply compass directions and scale. Communicate and interpret locations and directions, using compass directions, distances, and grid references.	

Holiday drop-offs FIO activity Copymaster	• Interpret a map to find the shortest route • Use mental strategies to add kilometres distances. • Make a distance map using a scale. Communicate and interpret locations and directions, using compass directions, distances, and grid references.	
Who lives where? FIO activity	Interpret a scale map Use a coordinate system or the language of direction and distance to specify locations and describe paths. *This activity is easier than required, but could be useful for students who need a bit more support as a starting point.*	
Map mysteries FIO activity Copymaster1 Copymaster2	Specify locations using grid references. Use a coordinate system or the language of direction and distance to specify locations and describe paths. *This activity is easier than required, but could be useful for students who need a bit more support as a starting point.*	

Activities that combine many ideas including location and navigation

Name of the activity (and link)	Description	Thumbnail of the activity
Day trippers FIO activity Copymaster	• Interpret information from a timetable. • Interpret information from a chart. • Interpret a scale map and draw a route. • Find average speeds. • Calculate percentages of money amounts. • Show percentages on a pie chart (awareness that this is no longer required, they could make a bar graph instead).	

Reflection activity (10 minutes)

Check in at the end

At the end of the lesson check in with the students on the content ideas for location and navigation from the start (slide 52) connecting the activities with the different location and navigation content areas.

Get students to reflect on their learning from the lesson, using the intended learning outcomes.

Resources

Activity resources

- Targeted Support for Numeracy Intervention Planning - Supplementary PowerPoint

Mathematics investigation cycle materials (available in other lessons)

- The Mathematics Investigation Cycle – poster and cards
- Mathematics investigation cycle (lesson 3)

Three reads maths language routine (MLR) (materials from other lessons)

- Operations on number (lesson 4) including three reads MLR
- Video about three reads MLR

Materials specific to this lesson

- Location problems (print enough for students working in small groups)
- Worked examples of location problems
- Figure it out activities (print as needed)

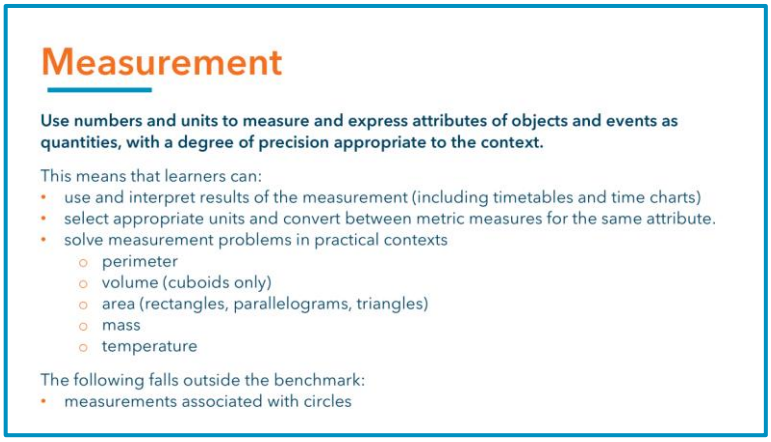
LESSON 8	**Measurement** (Co-operative learning)

Learning outcomes:

In this lesson, learners…

- are solving problems with a focus on *measurement*.
- are using cooperative learning approaches to solve these problems.

Learning activities

Time	
'Do now'/ Starter activity (10 minutes)	**Introduction and scene setting** Remind students about the content ideas for measurement (slide 53).  **Measurement** **Use numbers and units to measure and express attributes of objects and events as quantities, with a degree of precision appropriate to the context.** This means that learners can: • use and interpret results of the measurement (including timetables and time charts) • select appropriate units and convert between metric measures for the same attribute. • solve measurement problems in practical contexts ○ perimeter ○ volume (cuboids only) ○ area (rectangles, parallelograms, triangles) ○ mass ○ temperature The following falls outside the benchmark: • measurements associated with circles Explain that in this lesson they will be looking at questions that mostly contain measurement problems. The first lot of questions they will work with a partner, using think-pair-share, and then they will work in a slightly larger group working as numbered heads together.
Input/ knowledge-building activity (40 minutes)	**1. Using Think-Pair-Share to work on Numeracy CAA tasks (20-30 mins)** Remind them about the cooperative learning strategy of **Think-Pair-Share** that they were introduced to in the mathematical relationships lesson. 1. Students try the problem, they might like to use the three reads MLR to make notes about the problem and how they could solve it and if they can, they solve the problem. 2. Students pair up and both students share their ideas about how they solved the problem, each student explaining their method and solution, even if they are the same; or if they haven't solved it, what ideas did they have to try to solve it. Work together to find a solution. 3. Pairs are paired. One person from each pair shares their solution and methods with the other pair. The person who shares changes each time they pair up. Select the measurement problems from the initial session as listed in the table. Alternatively, this document has the six problems on it, or the Supplementary PowerPoint slides can be used.

Teachers might like to get the students to do the T2 2024 questions all together (think) and then pair and share, and then do the T3 2024 questions (think) and then pair and share. If you decide to do only one set, choose T3 2024 as that has an Outcome 3 example.

Content idea	T2 2024 Questions	T3 2024 Questions
Measurement	1a (Slide 9) - Outcome 1 1b (Slide 15) - Outcome 2 4e (Slide 13) - Outcome 1	2e (Slide 27) - Outcome 1 3a (Slide 29) - Outcome 1 5c (Slide 40) - Outcome 3

2. Using numbered heads together strategy to work on Numeracy CAA tasks (20-30 mins)

For these next activities rather than students thinking by themselves, then sharing, they are working from the start as a group on the problem using the **numbered heads together** strategy. Students may choose to work individually before coming to share their ideas, but this is not an expectation.

Remind the students that they will work together to find the solution to the problems given to them. The students in a group are numbered and then the teacher calls a number, and the student with that number shares the group's answer. This means that everyone in the group needs to understand the group's response.

There are three complete questions that students can work on using the numbered heads together strategy.

1. Measurement T2 2022 Q6a-b.pdf
    - (a) Outcome 1, measurement
    - (b) Outcome 1, measurement
    - Schedule
2. Measurement T2 2022 Q10.pdf
    - (a) Outcome 1, measurement
    - (b) Outcome 2, measurement
    - (c) Outcome 3, measurement
    - Schedule
3. Measurement T4 2022 Q9.pdf
    - (a) Outcome 1, measurement
    - (b) Outcome 2, spatial properties and representations
    - (c) Outcome 2, measurement
    - Schedule

Share one question at a time.

1. Members of the group work together to find the solutions to all parts of the question.
2. Allow time to solve the problem and then to make sure everyone in the group understands the solution.
3. Remind students that you will pick a number and that numbered person speaks on behalf of the group.

Reflection activity (10 minutes)

Check in at the end

At the end of the lesson check in with the students on the measurement content area ideas from the start (slide 53), connecting the activities with the different measurement content areas.

	Get students to reflect on their learning from the lesson, using the intended learning outcomes.

Resources

Activity resources

- Targeted Support for Numeracy Intervention Planning - Supplementary PowerPoint

Cooperative learning resources

- S4. Cooperative learning (think-pair-share & numbered heads together)

Materials specific to this lesson

- Measurement problems
- T2 2024 Numeracy assessment event 1 2024 paper.pdf | Schedule
- T3 2024 Numeracy assessment event 2 2024 paper.pdf | Schedule
- Measurement T2 2022 Q6a-b.pdf | Schedule
- Measurement T2 2022 Q10.pdf | Schedule
- Measurement T4 2022 Q9.pdf | Schedule

Additional supports

Observation of students during the lesson may identify gaps or areas needing further work.

Should you require them, here are some additional supports.

- In **noticing and acting** links are given to existing accelerating learning activities on Tāhūrangi. These supports primarily focus on area, perimeter and volume.
- The **Figure it out** resources also provide additional teaching and learning activities on measurement. These activities are in the style of the numeracy standard activities.

Noticing and acting

Notice where students might have difficulties with questions and identify if any of the accelerating learning activities might be useful to support knowledge building.

There are two groups of measurement activities in the accelerating learning materials. The main page has a set of diagnostic questions that can be used to identify student needs.

- Area and perimeter of rectangles (main page)
  - Finding the areas of rectangles using side lengths
    - The purpose of this activity is to support students to find areas of rectangles by measuring and multiplying side lengths.
  - Finding areas and perimeters from decimal side lengths
    - The purpose of this activity is to support students to find areas and perimeters of rectangle when the side lengths are decimal measurements.
  - Finding perimeters
    - The purpose of this activity is to support students to distinguish between area and perimeter of rectangles and measure the attributes in whole numbers of units, cm and cm^2.
  - Working with partial units
    - The purpose of this activity is to support students measure the areas and perimeters of the rectangle when the side lengths have simple fractions.

- Finding areas of rectangles
  - The purpose of this activity is to support students to identify rows and columns of units in rectangles and use multiplication to find areas.

- Volume and surface area of cuboids (main page)
  - Finding volumes of cuboids (whole number edge lengths)
    - The purpose of this activity is to support students to find the volumes of cuboids with whole number dimensions using multiplication.
  - Finding the volumes of cuboids by measuring edge lengths
    - The purpose of this activity is to support students in finding volumes of cuboids by measuring the edges in centimetres or interpreting diagrams in which the dimensions are given in whole numbers of centimetres. Students also use cubic centimetres (cm^3) to represent volume.
  - Finding surface area
    - The purpose of this activity is to support students to calculate surface areas of cuboids in an efficient manner using multiplication and addition.
  - Working with volume and surface area together
    - The purpose of this activity is to support students to distinguish between surface area and volume of cuboids, use multiplicative methods for finding the measurements, and record the measurements using appropriate units: square centimetres (cm^2) for surface area and cubic centimetres (cm^3) for volume. Students also need to develop a sense of size for important multiples of these units, such as 1000 cm^3 and 100 cm^2.
  - Finding volumes and surface areas from decimal side lengths
    - The purpose of this activity is to support students measure volumes and surface areas of cuboids when the side lengths are decimal measurements.

Figure It Out activities

Figure It Out books that provide a selection of measurement activities are:

Level 4 Measurement Book 1

Activities in this book

- Books in boxes | solving problems with mass and volume
- Castle construction | measuring length and area and constructing a scale model
- Cellphone confusion | working with rates and making comparisons
- Circle links | finding an approximate value of π from measurements (not in numeracy standard)
- Divide and prosper | drawing scale diagrams; working with large areas
- Energy crises | measuring and interpreting qualitative data
- Great gardens | exploring area, perimeter, and volume
- Hot stuff | measuring change against time

Level 4+ Measurement Book 2

Activities in this book

- Chilling out | constructing and using scales
- Colossal kiwifruit | working with circles, area, and volume (activity 2)
- Cylinder collection | comparing dimensions graphically (not in numeracy standard)
- Fat in foods | solving problems involving ratios and units of mass
- Gumboot games | working with distance, volume, mass, and rates of change
- Hot dogs | using ratio to compare surface area and volume
- Hot pots | performing calculations using 24-hour time
- Light and sound | measuring speed, using distance and time

- Icy contents | measuring and comparing volumes
- Little links | working with millimetres and applying circle formulae (not in numeracy standard)
- Petrol power | understanding the equivalence of units
- Scale of events | designing and using simple scales to measure qualitative data
- Taking off | solving problems with mass and volume
- Ways to go | interpreting and using timetables
- Weighty water | solving problems with mass and volume

Teachers notes

- On the right track | working with circles and drawing to scale (not in numeracy standard)
- Plentiful plankton | working with millimetres, micrometers, and ratios
- Pounamu pendants | estimating area and using percentages
- Round the bend | exploring the π relationship (not in numeracy standard)
- Television views | using ratio and/or scale drawing to find unknown lengths
- The big drip | working with length, area, and volume
- Time and tide | interpreting cyclical graphs

Teachers notes

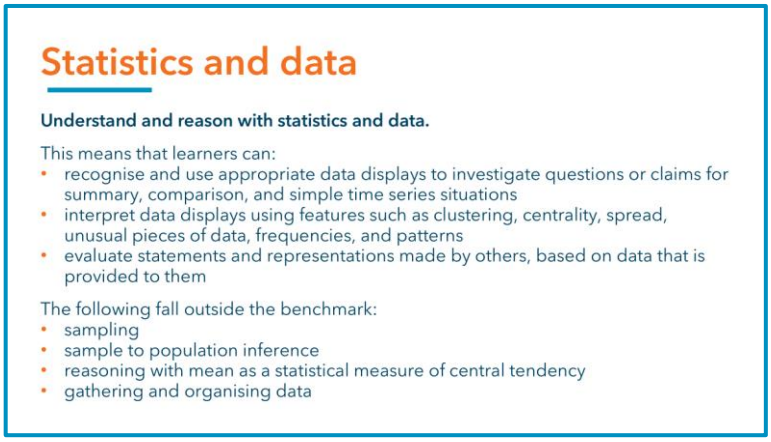
LESSON 9	**Statistics and data** (Statistics investigation cycle - PPDAC)

Learning outcomes:

In this lesson, learners…

- are solving problems with a focus on *statistics and data*.
- are using the statistical enquiry cycle to solve these problems.

Learning activities

Time	
'Do now'/ Starter activity (5 minutes)	**Introduction and scene setting** Remind students about the content ideas for elements of chance (slide 54),  **Statistics and data** **Understand and reason with statistics and data.** This means that learners can: • recognise and use appropriate data displays to investigate questions or claims for summary, comparison, and simple time series situations • interpret data displays using features such as clustering, centrality, spread, unusual pieces of data, frequencies, and patterns • evaluate statements and representations made by others, based on data that is provided to them The following fall outside the benchmark: • sampling • sample to population inference • reasoning with mean as a statistical measure of central tendency • gathering and organising data
Input/ knowledge-building activity (45 minutes)	**1. Outcome 1 & 2 short answer questions (15-20 mins)** 1.1 Hand out the Outcome 1 and 2 Statistics and data worksheet to students. 1.2 Working on one problem at a time, students try the problem. They might like to use the three reads MLR to make notes about the problem and how they could solve it and then solve the problem. 1.3 Work through all five problems, and once finished check in with their neighbour or group about their solutions. ○ Identify which questions they have different answers to and get them to share their different solutions explaining how they found the answer to see if they can get an agreed answer. 1.4 Share solutions with students, noticing any specific areas of difficulty. 1.5 Explicitly teach/attend to specific areas of difficulty. E.g., it could be to do with graph reading (pie graphs, bar graphs and time series are used); or it could be to do with reading scale, noticing percentages versus counts/frequency **2. Extended questions including outcome 3 (25-30 mins)** There are three complete questions that students can work on that include statistics and data. Depending on timing students may only get to do two of them, choose the order to share with students. You may like to use previous cooperative learning strategies such as **think-pair-share** or **numbered heads together**.

	While students are working on the tasks, identify different students (or groups) to share their solution for each part of the three questions, plus how they got it. Use the table below to note who you will ask.
	
	There are three complete questions that students can work on.
	• Statistics and data T2 2022 Q3.pdf
	  ○ (a) Outcome 1, Measurement
	  ○ (b) Outcome 2, Statistics and data
	  ○ (c) Outcome 3, Statistics and data
	  ○ Schedule
	
	• Statistics and data T2 2022 Q8.pdf
	  ○ (a) Outcome 1, Operations on number (finding the average)
	  ○ (b) Outcome 2, Statistics and data
	  ○ (c) Outcome 3, Statistics and data
	  ○ Schedule
	
	• Statistics and data T4 2022 Q10.pdf
	  ○ (a) Outcome 1, Statistics and data
	  ○ (b) Outcome 2, Statistics and data
	  ○ (c) Outcome 3, Statistics and data
	  ○ Schedule
	
	Allow time for students to share their solutions plus how they worked it out.

Reflection activity (10 minutes)	**Check in at the end** At the end of the lesson check in with the students on the statistics and data content area ideas from the start (slide 54), connecting the activities with the different statistics and data content areas. Get students to reflect on their learning from the lesson, using the intended learning outcomes.

Resources

Activity resources

- Targeted Support for Numeracy Intervention Planning - Supplementary PowerPoint

Statistical enquiry cycle

- Data Detective Poster - CensusAtSchool New Zealand

Materials specific to this lesson

- Outcome 1 and 2 Statistics and data worksheet
- T2 2024 Numeracy assessment event 1 2024 paper.pdf | Schedule
- T3 2024 Numeracy assessment event 2 2024 paper.pdf | Schedule
- T2 2023 Numeracy assessment event 1 2023 paper.pdf | Schedule
- T4 2023 Numeracy assessment event 2 2023 paper.pdf | Schedule
- T2 2022 Numeracy assessment event 1 2022 paper.pdf | Schedule
- T4 2022 Numeracy assessment event 2 2022 paper.pdf | Schedule

Table to identify who to share back

Question	Who to share solution
• Statistics and data T2 2022 Q3.pdf ○ (a) Outcome 1, Measurement	
○ (b) Outcome 2, Statistics and data	
○ (c) Outcome 3, Statistics and data	
• Statistics and data T2 2022 Q8.pdf ○ (a) Outcome 1, Operations on number (finding the average)	
○ (b) Outcome 2, Statistics and data	
○ (c) Outcome 3, Statistics and data	
• Statistics and data T4 2022 Q10.pdf ○ (a) Outcome 1, Statistics and data	
○ (b) Outcome 2, Statistics and data	
○ (c) Outcome 3, Statistics and data	

Additional supports

Statistics and data questions in recent numeracy CAA

Below is the list of accessible questions in recent numeracy CAA. A selection of these have been collated for this lesson. The list allows you to source other activities if you choose. The activities that are used in this lesson are highlighted in the tables.

Statistics and data in 2024 numeracy CAAs that can be used (i.e., contains no redacted images that impact on the question)

Content idea	T2 2024 Questions	T3 2024 Questions
Statistics and data	5f (Slide 21) - Outcome 3 Schedule	1b (Slide 23) - Outcome 1 2f (Slide 28) - Outcome 3 5a (Slide 38) - Outcome 1 Schedule

Statistics and data in 2023 numeracy CAAs that can be used (i.e., contains no redacted images that impact on the question)

Content idea	T2 2023 Questions	T4 2023 Questions
Statistics and data	2e - Outcome 2 5c - Outcome 3 Schedule	1h - Outcome 2 3d - Outcome 3 4e - Outcome 1 Schedule

Statistics and data in 2022 numeracy CAAs that can be used (i.e., contains no redacted images that impact on the question)

Content idea	T2 2022 Questions	T4 2022 Questions
Statistics and data	3b - Outcome 2 3c - Outcome 3 8b - Outcome 2 8c - Outcome 3 Schedule	2c - Outcome 3 4c - Outcome 3 7c - Outcome 3 8c - Outcome 3 10a - Outcome 1 10b - Outcome 2 10c - Outcome 3 Schedule

Figure It Out activities

Figure It Out books that provide a selection of statistics and data activities are:

Level 4 Statistics Book 1

Activities in this book

- Across the river | investigating the probability of an event
- Channel surfing | designing and implementing surveys and investigations
- Collect and reflect | investigating numeric data and presenting the findings
- Discipline dilemmas | reaching conclusions based on qualitative data
- Down the plug hole | collecting, interpreting, and communicating data
- Future options | investigating and interpreting events
- Mad minute | interpreting time-series graphs
- Monster munch | modelling situations and exploring natural variability
- Surf stats | interpreting time-series data and bar graphs
- What's the question? | analysing and writing survey questions

Teachers notes

Level 4+ Statistics Book 2

Activities in this book

- Action and reaction | comparing data and reaching conclusions
- Guess the mass | estimating mass; comparing distributions
- Just average | exploring how the average (mean) works
- Often absent | investigating data and reaching conclusions
- Populations pyramids | interpreting population pyramids
- Price hike | exploring percentage change using a time-series graph
- Social sounds | planning a statistical investigation
- Suspect on foot | estimating the value of one variable from the value of another
- Testing times | exploring bivariate data
- Walking tall? | comparing the shape of distributions
- Wim's waffles | looking for trends in time-series data

Teachers notes

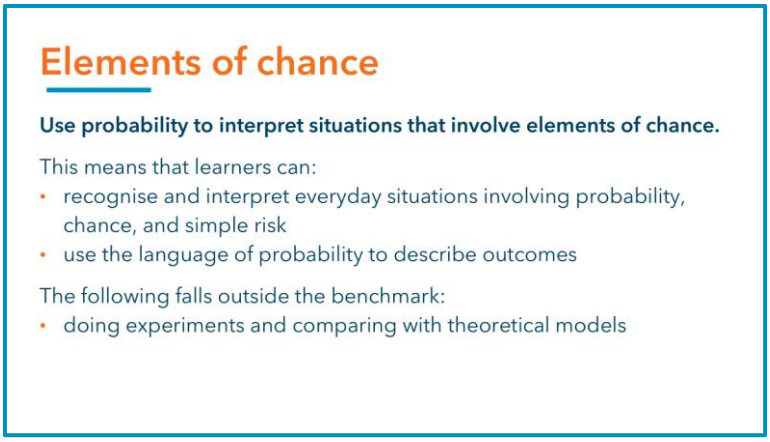
LESSON 10	**Elements of chance** (Mathematics language routines)

Learning outcomes:

In this lesson, learners…

- are solving problems with a focus on *elements of chance*.
- are introduced to the mathematical language routine of *stronger and clearer*.

Learning activities

Time	
'Do now'/ Starter activity (5 minutes)	**Introduction and scene setting** Remind students about the content ideas for elements of chance (slide 55).  **Elements of chance** **Use probability to interpret situations that involve elements of chance.** This means that learners can: • recognise and interpret everyday situations involving probability, chance, and simple risk • use the language of probability to describe outcomes The following falls outside the benchmark: • doing experiments and comparing with theoretical models
Input/ knowledge-building activity (45 minutes)	**1. Stronger and clearer - mathematical language routine (30-40 mins)** Initially students will explore three problems from the 2024 numeracy papers. All three problems require **elements of chance**. They will be introduced to the mathematical language routine (MLR) of **stronger and clearer** to help them to refine their ideas and language through structured peer interactions. 1.1 The three problems used are from the session on Understanding the Numeracy CAA, and include an example from Outcome 2 and two from Outcome 3. The three problems are:

Content idea	T2 2024 Questions	T3 2024 Questions
Elements of chance	4d (Slide 17) - Outcome 2 Schedule	3e (Slide 31) - Outcome 3 4f (Slide 37) - Outcome 3 Schedule

1.2 Introduce the activity to the students explaining that we are going to use a strategy called **stronger and clearer** to refine our ideas and language in our responses to problems. **Stronger and clearer** is one of the eight mathematical language routines (Zwiers et al., 2017)

1.2.1 Students think or write individually about a response to a problem, a first draft.

1.2.2 Use a structured pairing strategy to provide opportunities for students to refine and clarify their response through conversation (repeat with new partners).
1.2.3 Partners ask clarifying questions to deepen understanding.
1.2.4 Final written responses should reflect revisions and improved clarity.

1.3 Handout the problems to do. Teachers may choose to hand out one at a time, or all three together.

1.4 For the first problem (T2 2024 Q4d), ask students to do the problem individually, writing their response in the row, **first draft thinking**. Remind them that they might like to use the three reads MLR to make sense of the problem.
Encourage them to write their thoughts down, not just the answer.
Allow sufficient time for students to get something down.

1.5 Get the students to pair up and discuss their solutions, remind them to make notes from their discussion in **Share #1** so they don't forget. Allow 1-2 mins.

1.6 Now get the students to pair up with someone different, have further discussions about their solutions, including any ideas from the first pairing. Remind them to make notes from their discussion in **Share #2** so they don't forget. Allow 1-2 mins.

1.7 Students now rewrite their solution using information and ideas from their discussions in the **stronger and clearer** thinking row.

1.8 Invite 1 or 2 students to share their response.

1.9 Repeat approach for T3 2024 Q3e & Q4f.

2. Further elements of chance problems (balance of lesson)

There are five additional elements of chance questions set up to use. All five questions are available as:

- Write on sheets using the stronger and clearer thinking MLR

The questions on both sheets are outlined in the tables below.

Elements of chance in 2023 numeracy CAAs that can be used (i.e., contains no redacted images that impact on the question)

Content idea	T2 2023 Questions	T4 2023 Questions
Elements of chance	2a (can do without the images) - Outcome 1 Schedule	2e (can do without the image) - Outcome 3 4f (can do without the image) - Outcome 3 Schedule

Elements of chance in 2022 numeracy CAAs that can be used (i.e., contains no redacted images that impact on the question)

Content idea	T2 2022 Questions	T4 2022 Questions
Elements of chance	6c - Outcome 3 Schedule	3c - Outcome 3 Schedule

Reflection activity (10 minutes)

Check in at the end

At the end of the lesson check in with the students on the elements of chance content area ideas from the start (slide 55), connecting the activities with the different elements of chance content areas.
Get students to reflect on their learning from the lesson, using the intended learning outcomes.

Resources

Activity resources

- Targeted Support for Numeracy Intervention Planning - Supplementary PowerPoint

- S3. Mathematics language routines - Information about the mathematical language routines (MLR)
- Elements of chance - Stronger and Clearer student worksheet 1 (print one copy per student, check the lesson plan as to whether you want to print backed or single sided)
- Elements of chance - Stronger and clearer student worksheet 2 (print one copy per student, check the lesson plan to see which problems you want to do, and if you want printed backed or single sided)

Materials specific to this lesson

- T2 2024 Numeracy assessment event 1 2024 paper.pdf | Schedule
- T3 2024 Numeracy assessment event 2 2024 paper.pdf | Schedule
- T2 2023 Numeracy assessment event 1 2023 paper.pdf | Schedule
- T4 2023 Numeracy assessment event 2 2023 paper.pdf | Schedule
- T2 2022 Numeracy assessment event 1 2022 paper.pdf | Schedule
- T4 2022 Numeracy assessment event 2 2022 paper.pdf | Schedule

Additional supports

Should you require them, here are some additional supports.

- The Figure it out resources provide additional teaching and learning activities on elements of chance. These activities are in the style of the numeracy standard activities.

Figure It Out activities

Figure It Out books that provide a selection of elements of chance activities are:

Level 4 Statistics Book 1	Level 4+ Statistics Book 2
Activities in this book • Across the river | investigating the probability of an event • Catch of the match | finding and listing all possible combinations of events • Dodgy dice | determining unequal probabilities	Activities in this book • Birth months | finding probability by carrying out trials • Card sharp | finding the theoretical probability of events • On a plate | finding all possible permutations • Paper, scissors, rock | exploring probability by playing a game

• Family feast | finding and listing all possible combinations of events • Game show | exploring probability • Wallowing whales | exploring probability through a game • What's the chance? | estimating the probability of events Teachers notes	• Rough justice | finding the probabilities of compound events • Slater mazes | finding the probabilities of compound events • Unlucky lines | exploring probability based on area Teachers notes

Assessment capable learners

All leaners should be provided with multiple opportunities to engage with and practice using past assessment papers – both hardcopy and digital. This will foster greater assessment capability and digital readiness.

- Literacy and numeracy exemplars, past assessments, reports and schedules can be found here: Literacy and Numeracy past assessments and exemplars - NZQA
- Access to past digital exam papers can be found here: Find past digital external assessments » NZQA

Additional support resources

S1. Finding and using online accelerating learning supports in Tāhūrangi

There are some accelerated learning support materials on Tāhūrangi that could be used to identify and support identified gaps for students. These accelerated learning support materials have an initial diagnostic assessment page with 4-6 diagnostic questions. Each of the diagnostic questions then has a short teaching and learning sequence.

For example, a teacher might have identified an area needing support as percentages. They decide to use the diagnostic questions from Using percentages to represent part-whole relationships. From this they identify smaller groups with specific needs and they select some or all of the activities from within this collection.

- Simple fractions as percentages The purpose of this activity is to support students to develop their knowledge of fraction to percentage conversions.
- More complex fractions as percentages The purpose of this activity is to support students to develop their knowledge of fraction to percentage conversions. The problems are extended to include wholes that can be reduced to simple fractions using common factors.
- Scaling up to 100% The purpose of this activity is to support students to scale fractions to a percentage when the denominator is a factor or near factor of one hundred.
- Rounding to estimate percentages The purpose of this activity is to support students in developing their estimation skills related to expressing fractions as percentages.
- Finding the missing part The purpose of this activity is to support students to find the missing part in part-whole percentage problems.
- Finding the whole The purpose of this activity is to support students to find the missing whole in part-whole percentage problems.

A list of acceleration supports is below. These are sorted by content strand.

Number

- Fractions in ratios
    - Comparing ratios
    - Identifying fractions in simple ratios
    - Equivalent fractions in simple ratios
    - Comparing ratios with a common whole
    - Comparing ratios
- Putting decimal place value to work
    - Ordering decimals
    - Locating decimals on a scale
    - Adding and subtracting decimals
    - Multiplying decimals by 10 or 100
- Percentages as change (% can be more than 100%)
    - Simple discounts
    - Comparing in both directions
    - Percentage increases
    - More complex percentage increases
    - More complex discounts
- Using percentages to represent part-whole relationships
    - Simple fractions as percentages

  - More complex fractions as percentages
  - Scaling up to 100%
  - Rounding to estimate percentages
  - Finding the missing part
  - Finding the whole
- Rates – using multiplication and division
  - Unit rates
  - Applying multiplication and division to rates
  - Working with rates and fractions
  - Working with equivalent rates
  - Finding unit rates
- Integers

Measurement

- Area and perimeter of rectangles
- Volume and surface area of cuboids

Algebra

- Finding and expressing relationships (Level 4)

Statistics

- Working with numeric data

Possible useful readings from the ALiM materials

1: Fostering positive mathematical identities or PDF
2: Helping students develop the specialised language of mathematics or PDF
3: Supporting English language learners with the language of mathematics
4: Teaching mathematical language to English language learners or PDF
5: Helping students to participate in learning conversations or PDF
6: Building knowledge through questioning and modelling or PDF
7: Exploring measurement
8: Creating purposeful independent activities or PDF
9: Using ICT to support ALiM programmes
10: Shifting levels by developing multiplicative strategies
11: Addressing avoidance behaviours in mathematics classes
12: Challenging students with problems

S2. Productive mathematics discussion

> ***Students learn mathematics best when they are given opportunities to speak about mathematics using the language of mathematics***
>
> (Crillo, 2013, p.1).

Using the five practices to orchestrate productive mathematics discussion

The five practices for orchestrating productive mathematics discussions are anticipating (P1), monitoring (P2), selecting (P3), sequencing (P4) and connecting (P5) (Smith, & Stein, 2011).

- **Anticipating solutions - Practice 1**

  This is the first of the five practices for orchestrating productive mathematics discussions.

  Anticipating solutions involves carefully considering the various strategies students might use to approach a mathematical task. This includes both correct approaches and those that might lead to incorrect solutions due to errors or misconceptions.

  Anticipating solutions also means thinking about and noting potential responses to the work students are likely to produce. Preparing in this way allows teachers to plan actions and questions that guide students towards the lesson's goal without directing them down a fixed solution path. This proactive approach helps teachers avoid scrambling for ideas during the busy classroom environment. Russo (2020) refers to these as enabling prompts, which are useful for students struggling to make progress.

  Additionally, anticipating solutions involves identifying which strategies will best support the mathematics to be learned in the lesson. Making these decisions beforehand aids in the practices of monitoring and sequencing.

  A practical tip is to capture anticipated solution strategies in a table, categorising them into correct solutions and misconceptions. Include columns for noting which students used each strategy and for deciding the sequence of sharing these strategies. This table, referred to as the monitoring sheet, will be used throughout the lesson.

  As activities are conducted, new solution strategies that were not initially considered can be added, enriching the set of anticipated solutions over time. Collaborating with other teachers to explore different solution methods can also provide valuable insights.

- **Monitoring students' mathematical thinking & solution strategies - Practice 2**

  Successfully launching a task means all students are engaged in solving the problem. Enabling prompts or hints can support those needing further clarification. Once students are engaged, teachers can focus on monitoring their thinking as they work on the problem.

  Monitoring involves:
  - Listening to student conversations
  - Observing students' work
  - Keeping track of different approaches
  - Noting students who should report back
  - Asking questions to support progress or sense-making
  -

  Monitoring occurs while students work independently or collaboratively. The monitoring sheet prepared during planning helps teachers track who is using which strategy and make notes on student progress.

- **Selecting students to share their approaches - Practice 3**

  When planning a lesson, teachers identify key mathematical ideas they want students to learn. This drives task selection and the choice of students to share their solution strategies. Anticipated solutions and potential strategies to feature in the sharing part of the lesson are captured on the monitoring sheet.

  Teachers think about who will share their problem-solving approaches, **selecting** students whose strategies advance the class's mathematical understanding and align with the lesson's goals. Pre-selecting strategies ensures the mathematics at the heart of the lesson is discussed.

  Selecting students to share is crucial as it gives teachers control over the class discussion, ensuring the key mathematical concepts are addressed. Over time, all students should have opportunities to contribute and take centre stage.

- **Sequencing order for sharing student solution strategies - Practice 4**

  Sequencing involves determining the order in which student solutions are presented. While the sequence is often pre-determined during planning, unanticipated responses may require adjustments.

  Strategies can be sequenced in various ways:
  - From simple to complex
  - From most frequently used to least frequently used
  - From concrete to abstract

  Sometimes, addressing a misconception first can be beneficial. The chosen sequence should maximise opportunities for students to achieve the lesson's mathematical goals and develop powerful ideas.

- **Making connections to the learning goal - Practice 5**

  During discussions where students share solution strategies, teachers should find and use opportunities to connect to the mathematical learning goal. Ensuring the mathematics to be learned is explicitly addressed is crucial.

  At the start of student presentations, teachers should remind themselves of the lesson's learning goals. This helps them look for connections and opportunities to make the mathematical ideas public.

References

Ideas in this section based on:
Cirillo, M. (2013). What are some strategies for facilitating productive classroom discussions? *The National Council of Teachers of Mathematics*.
Russo, J. (2020). Designing and scaffolding rich mathematical learning experiences with challenging tasks. *Australian Primary Mathematics Classroom*, *25*(1), 3-10.
Smith, M. S., Bill, V., & Hughes, E. K. (2008). Thinking through a lesson: Successfully implementing high-level tasks. *Mathematics teaching in the middle school*, *14*(3), 132-138.
Smith, M. S., Hughes, E. K., Engle, R. A., & Stein, M. K. (2009). Orchestrating discussions. Mathematics Teaching in the Middle School, 14(9), 548-556.
Smith, M., & Stein, M. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: NCTM

Discourse-based instructional tools

- **Helping students clarify and share their thoughts**

  To effectively participate in discussions, students need to articulate their thinking and reasoning in a way that makes sense to others, or at least begins to.

  ### Wait Time

  Wait time is a strategy that allows students to think and gather their thoughts. During this time, the teacher remains silent. Ideally, wait time should be at least five seconds after asking a question. A helpful tip is to count slowly in your head, for example, "one Tongariro, two Tongariro, three Tongariro, four Tongariro, five Tongariro."

  When introducing wait time, explain to students that it can be frustrating when someone immediately calls out an answer before others have had time to think. To prevent this, let them know you will give them time to think—wait time—before asking for hands up. Emphasise that thinking time is crucial in mathematics and it's not about how quickly they find the answer.

  ### Turn and Talk

  Turn and talk, also known as think-pair-share, allows students to think about a problem individually and then discuss it with a partner. They can share their thoughts or practice what they might say to the whole class. Both partners get a chance to talk and listen. These discussions are usually brief, lasting about a minute.

  ### Say More

  "Say more" is a strategy to encourage students to provide more than a one or two-word answer. Simply ask them to elaborate. Initially, students might think their first response was wrong, so reassure them that you just want more information. Sometimes, asking them to say more helps clarify their thoughts.

  ### Revoicing

  Revoicing involves the teacher repeating some or all of what a student has said to verify understanding. This helps clarify the student's meaning and gives them a chance to correct or elaborate on their thoughts.

- **Helping students engage with others' thinking**

  Listening to others is a valuable skill for students. It allows them to hear different approaches to problems and learn the language of mathematics.

  ### Who Can Repeat?

  "Who can repeat?" is a strategy that encourages students to listen to others. It allows important contributions to be revisited, ensuring everyone hears and understands the idea. Repeating or rephrasing someone else's ideas helps students think about the concepts and connect them with their own ideas. Follow up by checking with the original student to see if the repetition or rephrasing was accurate.

  It takes time for students to understand that asking someone to repeat or rephrase an idea doesn't mean the idea was unclear or confusing, and it's not a test of their listening skills.

- **Helping students deepen their reasoning**

  Encouraging students to go beyond simply providing an answer helps them deepen their reasoning by requiring explanations or evidence for their responses.

  ### Why Do You Say That?

  "Why do you say that?" is a powerful talk move, not just for mathematics, but for life. It encourages students to delve into their thought processes, learn to reason, provide evidence, and justify their answers. Responses like "just because" are not sufficient. As teachers, we need to push for deeper reasoning by asking questions such as, "Why do you think that?" or "What evidence supports your answer?" Initially, students might feel they are wrong when asked to explain their reasoning, so it's important to reassure them that the goal is to understand their thought process better.

- **Helping students engage with the reasoning of others**

  Taking students beyond justifying their own understanding, we encourage them to reason and discuss both their ideas and those of others. This fosters truly productive mathematical discussions.

  ### Do You Agree or Disagree? And Why?

  "Do you agree or disagree? And why?" is a talk move that allows students to express their agreement or disagreement with a previously stated solution or claim, and support their position. It's crucial for students to learn to agree or disagree with the "claim" rather than the person. Since students might feel uncomfortable disagreeing with someone, it's helpful to practice phrasing it differently.

  For example, instead of saying, "I disagree with Sifa," students can say, "I disagree with Sifa's claim that adding two odd numbers results in an odd number. I believe that adding two odd numbers gives an even number because each odd number is one more than an even number. When you add the extra ones together, you get two, which is even, so the total is even."

  ### Who Can Add On?

  "Who can add on?" is a talk move that encourages students to engage with and expand on others' ideas. It invites students to build upon an explanation that another student has started, fostering a collaborative learning environment.

References

Ideas in this section based on:
Chapin, S. H., & O'Connor, C. (2007). Academically productive talk: Supporting students' learning in mathematics. *The learning of mathematics*, *69*, 113-128.
Chapin, S. H., O'Connor, C., & Anderson, N. C. (2013). Classroom discussions in Math: a teacher's guide for using talk moves to support Common Core and more. *Scholastic Inc*.
Cirillo, M. (2013). What are some strategies for facilitating productive classroom discussions? *The National Council of Teachers of Mathematics*.

S3. Mathematics language routines

The mathematical language routines (MLRs) provide guidance to mathematics and statistics teachers to enhance students' understanding of mathematical and statistical concepts while improving their language skills. These routines are adaptable across year levels and emphasise meaningful communication. The MLRs were developed by a group from Stanford University: Jeff Zwiers, Jack Dieckmann, Sara Rutherford-Quach, Vinci Daro, Renae Skarin, Steven Weiss, and James Malamut.

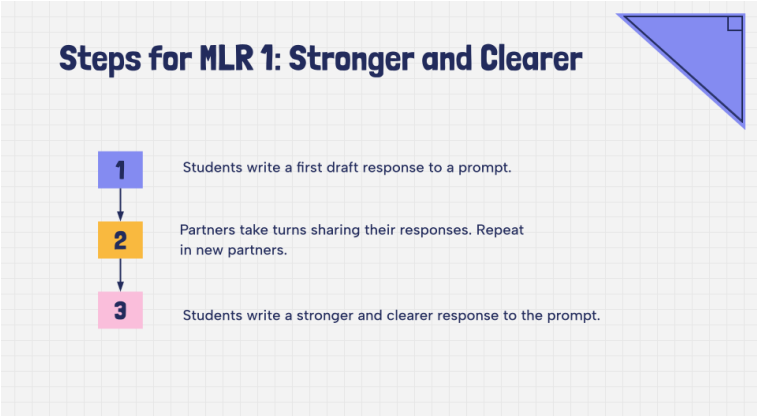
The **eight mathematical language routines** are listed below, and there is a summary of each one following. For detail on specific mathematical language routines please see Principles for the design of mathematics curricula: Promoting language and content development paper.

MLR1: Stronger and Clearer Each Time
MLR2: Collect and Display
MLR3: Clarify, Critique, and Correct
MLR4: Information Gap
MLR5: Co-Craft Questions
MLR6: Three Reads
MLR7: Compare and Connect
MLR8: Discussion Supports

Mathematical Language Routine 1: Stronger and Clearer Each Time

This routine encourages students to refine their ideas and language through structured peer interactions. It involves pre-writing, sharing, and revising responses based on feedback from partners.

- Students write initial thoughts on a problem.
- They engage in successive pair shares to clarify and strengthen their ideas.
- Partners ask clarifying questions to deepen understanding.
- Final written responses should reflect revisions and improved clarity.



Mathematical Language Routine 2: Collect and Display

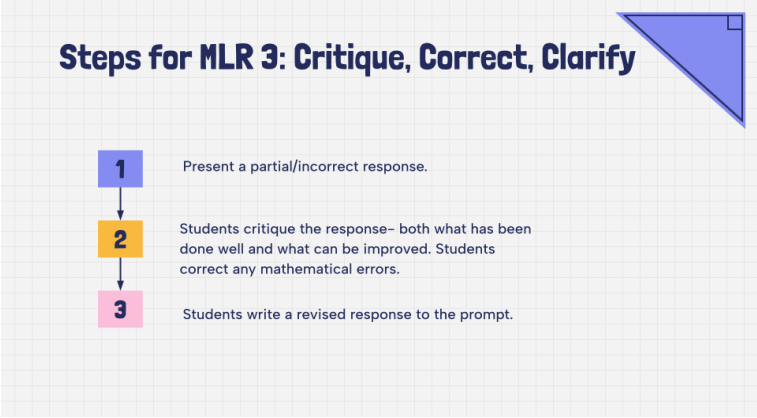
The purpose of this routine is to capture and display students' language during discussions, creating a reference for future use. It helps stabilise language and supports meta-awareness.

- Teachers listen and scribe student language during discussions.
- Collected language is displayed for reference in future lessons.
- Students reflect on the usefulness of displayed language in communication.

Mathematical Language Routine 3: Critique, Correct, and Clarify

This routine involves analysing and improving flawed mathematical writing. It promotes critical thinking and reflection on mathematical arguments.

- Students critique a partial or flawed response.
- They identify errors and ambiguities in the writing.
- Pairs share improved responses and refine their drafts.



Mathematical Language Routine 4: Information Gap

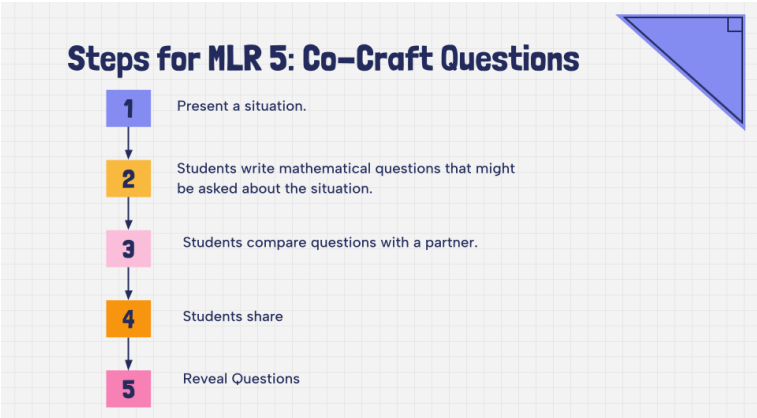
This routine creates a need for communication by providing partners with different pieces of information necessary to solve a problem. It fosters meaningful interactions.

- Partners have distinct information that must be shared to solve a problem.
- Students ask specific questions to gather necessary information.
- Explanations are provided to justify the use of shared information.

Mathematical Language Routine 5: Co-Craft Questions and Problems

This routine allows students to generate and analyse mathematical questions and problems collaboratively. It encourages deeper engagement with mathematical concepts.

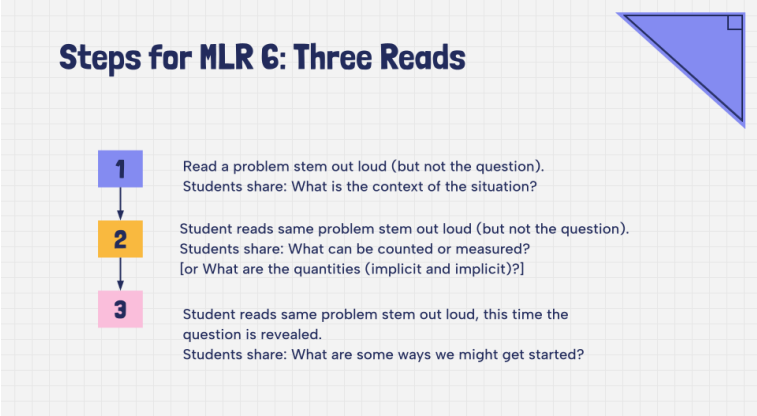
- Students write possible questions based on a presented situation.
- They compare and share their questions in pairs.
- Actual questions for further work are revealed after discussion.



Mathematical Language Routine 6: Three Reads

This routine supports reading comprehension and negotiation of meaning by having students read a problem three times with different focuses.

- The first read focuses on comprehension without numbers.
- The second read analyses the language used in the problem to present the mathematical structure.
- The third read brainstorms possible solution methods.



Mathematical Language Routine 7: Compare and Connect

This routine fosters meta-awareness by having students compare different mathematical approaches and representations. It encourages reflection on various strategies.

- Students identify similarities and differences in solution strategies.
- They focus on specific mathematical relationships and operations.
- Discussions prompt students to articulate their reasoning and understanding.

Mathematical Language Routine 8: Discussion Supports

This routine enhances classroom discussions by providing strategies for inclusive and rich conversations about mathematical ideas.

- Teachers revoice student ideas to clarify and involve others.
- Probing questions encourage deeper explanations and examples.
- Multi-modal strategies are used to engage students in discussions.

Resources

Stanford University

- Principles for the Design of Mathematics Curricula and MLRs (webpage)
- Principles for the design of mathematics curricula: Promoting language and content development (paper)

Auckland Maths Association | AMA online videos and presentations

- Keely Macher-Wessels | Get stronger and clearer student thinking | 5 November 2022
  - Looks at MLR1: Stronger and Clearer Each Time and MLR6: Three Reads
- Keely Macher-Wessels | More maths language routines | 10 June 2023
  - Looks at MLR3: Critique, Correct, and Clarify and MLR5: Co-Craft Questions and Problems
- MLR1, MLR3, MLR5 and MLR6 slides also shared with permission from Keely

S4. Cooperative learning

Cooperative learning is an instructional strategy where small groups of students work together on a common task. Each member is responsible for their own learning as well as helping their teammates learn, fostering a collaborative and supportive learning environment.

Here are some examples of cooperative learning strategies that can be used in mathematics and statistics classes:

1. Think-Pair-Share (TPS)

Think, Pair, Share is a way of providing increased wait time so that students have time to think before they speak or write. It also provides opportunities for repetition and for practising and polishing language. This strategy is very easy to use and takes no preparation and it can be used across all learning areas.

The students first work individually writing down their ideas. Next they share their ideas with a partner and then with a larger group or whole class. It is important that students need to be able to share their partner's ideas as well as their own.

The 'wait or think' time that is part of Think, Pair, Share has been demonstrated to be a powerful factor in improving student responses to questions.

From ESOL online: https://esolonline.tki.org.nz/ESOL-Online/Planning-for-my-students-needs/Resources-for-planning/ESOL-teaching-strategies/Oral-Language/Speaking-strategies/Think-pair-share

Example

Students might individually consider how to interpret a dataset, then discuss their interpretations with a partner, and finally present their conclusions to the class.

2. Jigsaw

Jigsaw is a cooperative learning strategy that enables each student of a "home" group to focus on one area, idea or 'piece of the puzzle'. Students become 'experts' in their assigned area by working with members from other groups who are also assigned that same focus. These new groups are called 'expert' groups. The 'experts' then return to their 'home' group and share their expertise, which is required for the completion of the home group task. As in a jigsaw puzzle, each piece or each student's part is essential to enable the completion and understanding of the home group task.

The strategy can be used across most curriculum levels, once students are able to work with some autonomy in a group.

From NZ curriculum online:
https://nzcurriculum.tki.org.nz/content/download/139998/1043535/file/Jigsaw%20-%20cooperative%20learning.doc

Example

When learning about properties of 2D shapes, one student might learn about the properties of triangles, another about quadrilaterals, and another about circles. They then teach their respective topics to their group members.

3. Group Investigation

Students work in small groups to investigate a topic, plan their research, and present their findings.

Example

Students engage in mathematics investigations or statistics investigations in small groups. They undertake their investigation and present their findings.

4. Numbered Heads Together

Students in a group are numbered. The teacher asks a question or provides a problem to solve, and students put their heads together to discuss the answer. The teacher then calls a number, and the student with that number shares the group's answer.

See more at Numbered Heads Together - Active Learning Strategies

Example

In a mathematics class, the teacher might ask a question about finding the area of a composite shape. Students discuss the solution, and the student with the called number explains the group's solution.

5. Peer Tutoring

Peer tutoring is a form of cooperative learning where two students work together – a more skilled "tutor" with a less skilled "tutee".

From Inclusive Education https://inclusive.tki.org.nz/guides/supporting-positive-peer-relationships/cooperative-learning-strategies/

Example

In class, a student who understands how to find the volume of a prism might tutor a peer who is struggling with the concept, explaining the steps and working through problems together.

Further readings

An introduction to cooperative learning - THE EDUCATION HUB
Strategies for teaching and learning in Social Studies - Cooperative learning

[i] Ministry of Education, (2207), *New Zealand Curriculum*, retrieved from: https://newzealandcurriculum.tahurangi.education.govt.nz/new-zealand-curriculum/5637175326.p (19/06/25)
[ii] Smaill, E., and Darr, C., (2020), *An examination of the curriculum-levelling construct,* retrieved from: https://www.nzcer.org.nz/sites/default/files/downloads/An%20Examination%20of%20the%20Curriculum-Levelling%20Construct.pdf (22/04/25)
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[v] Ministry of Education, (n.d.), *What is numeracy about?*, retrieved from: https://ncea.education.govt.nz/literacy-and-numeracy/numeracy/learning
[vi] Ministry of Education, (2024), *Accelerating progress in literacy – teacher guidance*, retrieved from: Accelerating progress in literacy – Teacher guidance (23/04/25)
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[viii] Ministry of Education, (n.d.), NCEA Co-requisite (website), https://ncea.education.govt.nz/overview-NCEA-corequisite-standards#:~:text=happening%20from%202024?-,All%20learners%20need%20to%20achieve%20a%2020%2Dcredit%20co%2Drequisite,to%20gain%20an%20NCEA%20qualification. (28/04/25).
[ix] Ministry of Education, (n.d.), Weave diagram for numeracy illustrating process and content ideas, retrieved from: https://ncea.education.govt.nz/literacy-and-numeracy/numeracy/learning